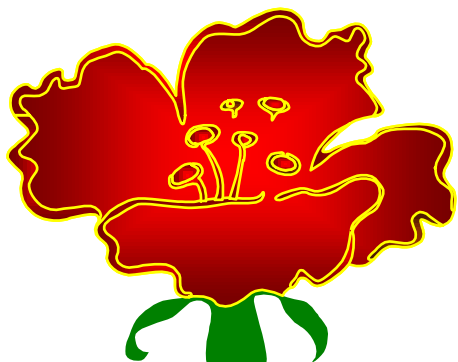


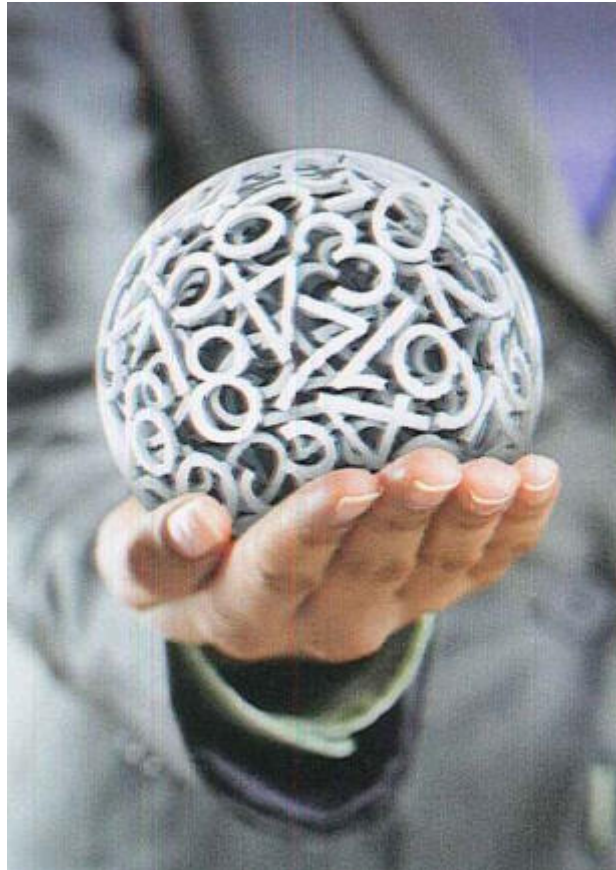
Mathematics

Preparatory one

Second Term



Unit 1



Unit lessons:

- 1) Powers and exponents
- 2) Scientific notation
- 3) Square roots and Cube roots

Lesson (1)

Powers and Exponents

Repeated Multiplication and Exponential form

The result of multiplying repeated factors can be expressed using the powers or exponential form, that has an exponent and a base.

For example :

A factor repeated 4 times

$$3 \times 3 \times 3 \times 3 = 3^4$$

The exponent : indicates the number of times the base is used as a factor.

The base : is the repeated factor.

It is read as "3 raised to the power of 4" or "3 to the power of 4", meaning "3 multiplied by itself 4 times".

Example ①

Write each of the following using exponents :

1 $7 \times 7 \times 7$

2 $(-2) \times (-2) \times (-2) \times (-2) \times (-2)$

3 $3 \times 3 \times a \times a \times a$

4 $(-x) \times (-x) \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times (-x) \times (-x)$

• Solution :

- 1-
 2-
 3-
 4-

2) Find the value of the following :

1- 2^3

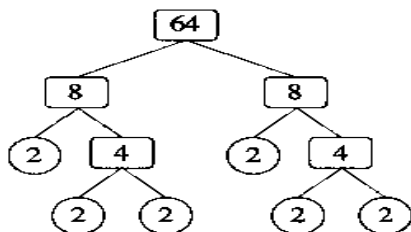
2- $(-\frac{1}{2})^5$

• Solution :

- 1-
 2-

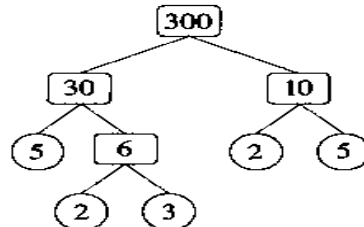
3) Factorize the numbers into its prime factors as follows :

1



$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

2



$$300 = 5 \times 5 \times 2 \times 2 \times 3 = 5^2 \times 2^2 \times 3$$

4) Factorize the numbers into its prime factors :

1- 81

2- 125

• Solution :

.....

.....

.....

Even and odd exponents of a negative base

If a is a positive number , and m is a positive integer , then :

$$(-a)^m = a^m$$

when m is an **even** number

For example : $(-3)^2 = 3^2 = 9$

$$(-a)^m = -a^m$$

when m is an **odd** number

For example : $(-3)^3 = -3^3 = -27$

• Notice the difference between : $(-3)^2$ and -3^2

Where $(-3)^2 = 9$ but $-3^2 = -9$

5) Find in simplest form each of the following :

1- $(-5)^3$

2- $(-5)^2$

• Solution :

1-

2-

6) If $a = 3$, $b = -1$, $c = 2$, find the numerical value of each :

1- $a^2 - 2ab$

2- $(a - b)^c$

• Solution :

1-

2-

Multiplication and division of powers with the same base

Multiplication Law

$$3^2 \times 3^3 = \overbrace{(3 \times 3)}^{\text{Factors}} \times \overbrace{(3 \times 3 \times 3)}^{\text{Factors}} = 3^5$$

This means that when multiplying powers with the same base , keep the base and add the exponents.

In general

For any rational number a and two

integers m, n : $a^m \times a^n = a^{m+n}$

For example : $7^8 \times 7^5 \times 7 = 7^{8+5+1} = 7^{13}$

Division Law

$$\frac{7^7}{7^5} = \frac{\overbrace{7 \times 7 \times 7 \times 7 \times 7}^{\text{Factors}}}{\overbrace{7 \times 7}^{\text{Factors}}} = 7 \times 7 \times 7 = 7^3$$

This means that when dividing powers with the same base , keep the base and subtract the exponents.

In general

For any rational number a , not equal to zero ,

and two integers m, n : $\frac{a^m}{a^n} = a^{m-n}$

For example : $8^6 \div 8^4 = 8^{6-4} = 8^2$

7) Choose the correct answer :

A) $7^2 \times 7^3 = \dots\dots\dots$

(a) 7

(b) 7^5

(c) 7^6

(d) 7^7

B) $5^8 \div 5^6 = \dots\dots\dots$

- (a) 1^2 (b) 5^4 (c) 5^2 (d) 1^4

C) $3^5 \times \dots\dots\dots = 3^{10}$

- (a) 3^2 (b) 3^5 (c) 3^{15} (d) 3^3

D) $\dots\dots\dots \div 8^2 = 8^3$

- (a) 8^9 (b) 8^6 (c) 8^5 (d) 8^1

E) Half of the number 2^6 is

- (a) 2^3 (b) 1^6 (c) 1^3 (d) 2^5

8) Complete each of the following:

1- $3^a \times \dots\dots\dots = 3^{a+b}$

2- $x^6 \div \dots\dots\dots = x^2$

9) Find in simplest form the result of the following:

1 $\frac{8^4 \times 8^6}{8^3 \times 8^5}$

2 $\left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^5 \times \left(\frac{4}{5}\right)$

3 $\frac{(-3)^6 \times 6^7}{6^5 \times (-3)^3}$

4 $\frac{(-x)^6 \times x^3}{(-x)^5 \times (-x)^2}$

Solution

1 $\frac{8^4 \times 8^6}{8^3 \times 8^5} = \frac{8^{4+6}}{8^{3+5}} = \frac{8^{10}}{8^8} = 8^{10-8} = 8^2 = 64$

Another solution for number 1

$$\frac{8^4 \times 8^6}{8^3 \times 8^5} = 8^{4+6-3-5} = 8^2 = 64$$

2 $\left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^5 \times \left(\frac{4}{5}\right) = \left(\frac{4}{5}\right)^{7-5+1} = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

3 $\frac{(-3)^6 \times 6^7}{6^5 \times (-3)^3} = (-3)^{6-3} \times 6^{7-5} = (-3)^3 \times 6^2 = -27 \times 36 = -972$

4 $\frac{(-x)^6 \times x^3}{(-x)^5 \times (-x)^2} = \frac{x^6 \times x^3}{-x^5 \times x^2} = \frac{x^{6+3}}{-x^{5+2}} = \frac{x^9}{-x^7} = -x^{9-7} = -x^2$

Zero exponent and negative integer exponents:

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

$\xrightarrow{\div 3}$ $\xrightarrow{\div 3}$ $\xrightarrow{\div 3}$ $\xrightarrow{\div 3}$ $\xrightarrow{\div 3}$ $\xrightarrow{\div 3}$

It is observed from the previous pattern that :

• $3^0 = 1$ • $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9} = \frac{1}{3^2}$, $3^{-3} = \frac{1}{27} = \frac{1}{3^3}$

In general

1 Any number , not equal to zero , raised to the power of zero equals to 1

i.e. $a^0 = 1$ where $a \neq 0$

For example : $7^0 = 1$, $\left(-\frac{2}{3}\right)^0 = 1$

2 Any number , not equal to zero , raised to the power of $(-n)$ is equal to the multiplicative inverse of the same number raised to the power of n .

i.e. $a^{-n} = \frac{1}{a^n}$ where $a \neq 0$

For example : $3^{-2} = \frac{1}{3^2}$, $\left(\frac{7}{8}\right)^{-3} = \left(\frac{8}{7}\right)^3$, $\left(\frac{5}{9}\right)^{-1} = \frac{9}{5}$, $\left(\frac{1}{3}\right)^{-2} = 3^2$

10) Simplify each of the following to its simplest form :

1) $(-2)^{-3}$

2) $2^4 \times 2^{-2}$

3) $\frac{5^{-2}}{5^{-3}}$

4) $\frac{6^{-3} \times 6^5}{6^2}$

5) $\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}$

6) $\frac{x^{-4} \times x^{-3}}{x^{-2} \times x^{-1}}$

• Solution :

- 1-
 2-
 3-
 4-
 5-
 6-

Homework**(1) Write the following using exponents:**

a) $2 \times 2 \times 2 \times 2 ?$

b) $b \times 5 \times b \times 5 \times b ?$

(2) If : $a = -3$, $b = 4$, then find the numerical value of each :

1- a^b

2- $(a + b)^2$

3) Choose the correct answer :

A) $(-2)^3 = \dots\dots\dots$

(a) - 6

(b) 6

(c) 8

(d) - 8

B) The additive invers of 4^{-3} is

(a) $(-4)^3$

(b) $(-4)^{-3}$

(c) 4^3

(d) 4^{-3}

C) $5a^0 - (5a)^0$ is

(a) 0

(b) 4

(c) 5

(d) 10

D) Third of the number 3^x is

(a) 1^x

(b) 3^{x-1}

(c) 3^{x+1}

(d) 1^5

Lesson (2)

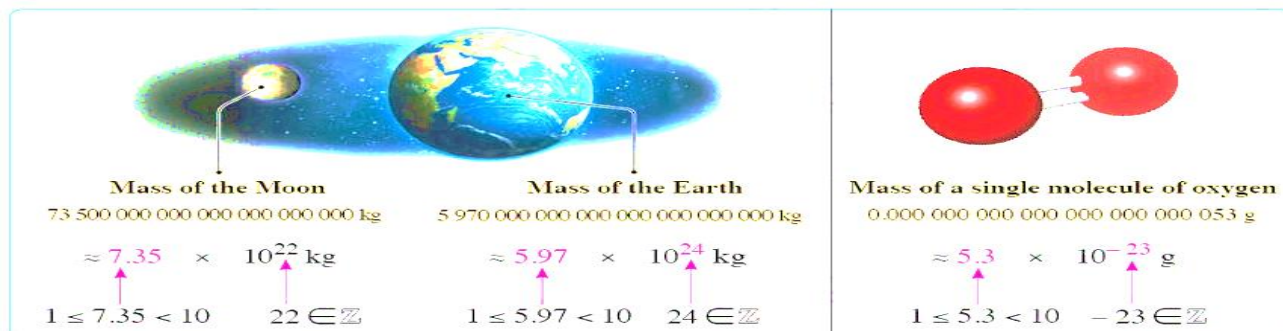
Scientific Notation

Scientific notation is a method of writing very large or very small numbers, where the number is expressed as the product of two factors, one of them has an absolute value greater than or equal to 1 and less than 10, and the other is 10 raised to an integer power.

i.e. The number is expressed in scientific notation as :

$$a \times 10^n \text{ where } 1 \leq |a| < 10, n \in \mathbb{Z}$$

The following image shows some very large and very small masses written in scientific notation:

**Example ①**

Which of the following numbers is written in scientific notation and which is not ?
And why?

1 4.6×10^8

2 45×10^8

3 -9.6×10^{10}

4 0.72×10^{-6}

5 1×10^{-7}

6 $-3.7 \times 10^{2.5}$

2) Write each of the following in scientific notation :

1- 82 000 000

2- 0.00000135

2- 125×10^8

• Solution :

1-

2-

3-

3) Write each of the following in standard form :

1- 3.7×10^5

2- 7.38×10^{-3}

• Solution :

1-

2-

4) The following table illustrates the distance of planets from The sun in Kilometers:

Planet	Saturn	Earth	Mars	Uranus
Distance from the Sun	1.43×10^9	1.5×10^8	2.28×10^8	2.87×10^9

Arrange the planets from closest to farthest from the Sun.

• Solution :

..... , , ,

Operations on numbers in scientific notation :Addition and subtraction in scientific notation :

(5) Find the result of the following in scientific notation :

1- $(3.7 \times 10^5) + (2.3 \times 10^6)$ 2- $(0.2 \times 10^8) - (3.2 \times 10^9)$

• Solution :

1-

2-

Multiplication and division in scientific notation :

(6) Find the result of the following in scientific notation :

1- $(6.5 \times 10^4) \times (8 \times 10^2)$ 2- $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4})$

• Solution :

1-

2-

Homework

1) Write the following in scientific notation:

a) 4 650 000 ?

b) 192 000 000 ?

c) 5.3×10^7 ?

d) 0.023×10^8 ?

e) 0.000000021 ?

f) 0.0000543 ?

g) 12 million ?

h) 0.25 milliard ?

2) Write each of the following in standard form :

1- 2.4×10^4

2- 4.76×10^{-2}

3) Arrange in a descending order :

A) 16×10^{-6} , 1.5×10^{-5} , 0.8×10^{-5} , 14×10^{-4} ?

• Solution :

..... , , ,

4) Choose the correct answer :**A) is a scientific notation**

- (a) $1.5 \times 10^{4.5}$ (b) 31.5×10^5 (c) 15×10^5 (d) 3.14×10^5

B) $0.000073 = \dots\dots\dots$

- (a) 17.3×10^6 (b) 7.3×10^5 (c) 7.3×10^{-5} (d) 7.3×10^{-6}

C) $800\,000 = 8 \times 10^m$, the value of m =

- (a) 4 (b) - 4 (c) 5 (d) - 5

D) $0.000375 = 3.75 \times 10^m$, the value of m =

- (a) 4 (b) - 4 (c) 5 (d) - 5

**e) If the number $y \times 10^{-9}$ is written in scientific notation,
what could be the value of y ?**

- (a) 0.6 (b) 6 (c) 60 (d) 600

**F) If the speed of the light is $300\,000\text{ km/s}$, what is the speed
of light in m/s ?**

- (a) 3×10^5 (b) 3×10^7 (c) 3×10^8 (d) 3×10^{10}

5) Arrange in an ascending order :

A) 7.3×10^{11} , 6.9×10^{12} , 0.537×10^{13} ?

• Solution :

..... , ,

**6) The temperature at the center of the sun is 15 million
Celsius. Write this temperature in scientific notation?**

Lesson 3: Square Roots and Cube Roots

- The product of a number multiplied by itself is the square of that number.

For example : the number 16 is the square of the number 4

because $4 \times 4 = 16$

- Squares of whole numbers :

$1^2, 2^2, 3^2, 4^2, 5^2, \dots$

are equal to :

1, 4, 9, 16, 25, ... and these are perfect squares.

- The product of a number multiplied by itself three times is the cube of that number.

For example : the number 64 is the cube of the number 4

because $4 \times 4 \times 4 = 64$

- Cubes of whole numbers :

$1^3, 2^3, 3^3, 4^3, 5^3, \dots$

are equal to :

1, 8, 27, 64, 125, ... and these are perfect cubes.

The Square Root of a Perfect Square Number

- The square root of a perfect square number (a) is the number whose square equals (a).

That is : the square root of a number means finding the number which, when multiplied by itself, gives that number.

- A perfect square number has two square roots, one is positive and the other is negative, and each of them is the additive inverse of the other.

For example : the number 16 has two square roots which are -4 and 4

Because : $(-4)^2 = 16$ and $(4)^2 = 16$

- The symbol $\sqrt{\quad}$ denotes the positive square root of a number.



Note that

- $\sqrt{0} = 0$
- It is meaningless to find $\sqrt{a}$ if the number a is negative, because there is no number when multiplied by itself, it results a negative outcome.

Example 1

Complete each of the following :

1 $\sqrt{36} = \dots\dots\dots$

2 $-\sqrt{\frac{16}{25}} = \dots\dots\dots$

3 $\pm\sqrt{0.25} = \dots\dots\dots$

4 The two square roots of the number 49 are $\dots\dots\dots$ and $\dots\dots\dots$

5 The positive square root of the number $\frac{9}{64}$ is $\dots\dots\dots$

6 The negative square root of the number 36 is $\dots\dots\dots$

**Notes**

① $\sqrt{a^2} = |a|$

For example : $\bullet \sqrt{(-3)^2} = |-3| = 3$

$$\bullet \sqrt{\left(-\frac{2}{9}\right)^2} = \left|-\frac{2}{9}\right| = \frac{2}{9}$$

② $\sqrt{a^{2n}} = |a^n|$ where n is an integer

For example : $\bullet \sqrt{a^6} = |a^3|$

$$\bullet \sqrt{a^4} = |a^2| = a^2$$

Example ②

Express each of the following in its simplest form :

1 $\pm \sqrt{2 \frac{7}{9}}$

2 $\sqrt{\left(-\frac{2}{7}\right)^2}$

3 $\sqrt{16 + 9}$

4 $\sqrt{4 \times 25}$

5 $\sqrt{\frac{9x^4}{100y^{20}}}$

6 $\sqrt{\frac{36a^6}{49}}$

Solving Equations Using the Square Root

- If $x^2 = a$ where $a \geq 0$, then $x = \pm \sqrt{a}$ For example : If $x^2 = 81$, then $x = \pm \sqrt{81} = \pm 9$
- If $\sqrt{x} = a$, then $x = a^2$ For example : If $\sqrt{x} = 3$, then $x = 3^2 = 9$

Example ③Find the value of x in each of the following :

1 $\sqrt{x} + 2 = 4$

2 $x^2 + 1 = 17$

3 $2x^2 - 4 = 46$

**Solution**

1 $\therefore \sqrt{x} + 2 = 4$

$$\therefore \sqrt{x} = 4 - 2 = 2$$

$$\therefore x = 2^2 = 4$$

2 $\therefore x^2 + 1 = 17$

$$\therefore x^2 = 17 - 1 = 16$$

$$\therefore x = \pm \sqrt{16} = \pm 4$$

3 $\therefore 2x^2 - 4 = 46$

$$\therefore 2x^2 = 46 + 4 = 50$$

$$\therefore x^2 = \frac{50}{2} = 25$$

$$\therefore x = \pm \sqrt{25} = \pm 5$$

Example ④

A cube has a total (surface) area of 150 square centimeters. Calculate its volume.



Exercises(1)

Complete each of the following :

1 $\sqrt{\frac{16}{9}} \times \frac{3}{4} = \dots\dots\dots$

2 $\sqrt{\dots\dots\dots} = 9$

3 $\sqrt{4} - \sqrt{16} = \dots\dots\dots$

4 The multiplicative inverse of the number $\sqrt{\frac{4}{25}}$ in its simplest form is $\dots\dots\dots$

5 The additive inverse of the number $\sqrt{36x^4}$ is $\dots\dots\dots$

6 If the area of a square is 64 square centimeters , then its side length = $\dots\dots\dots$ cm.

7 $\sqrt{3 + \dots\dots\dots} = 7$

8 $\sqrt{3 \times \dots\dots\dots} = 9$

Write each of the following in its simplest form :

1 $\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^0 \times \left(-\frac{2}{7}\right)^2$

2 $\frac{2}{5} \times \sqrt{\frac{49}{16}} \div \left(-\frac{1}{2}\right)^3$

3 $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{4}\right)^0$

Find the value of x in each of the following :

1 $\sqrt{x} = 9$

2 $\sqrt{x+1} = 7$

3 $x^2 = 121$

4 $x^2 - 1 = 8$

5 $3x^2 - 5 = 43$

6 $9x^2 + 4 = 29$

7 $7x^2 - 5 = 5x^2 + 13$

8 $2(x^2 - 3) = x^2 + 3$

Find the solution set for each of the following equations in $\mathbb{Z}$:

1 $x^2 = 25$

2 $3x^2 - 2 = 25$

3 $2x^2 + 1 = 33$

4 $\frac{1}{2}x^2 - 3 = 5$

5 $6x^2 - 2 = 4x^2 + 6$

6 $4(x^2 - 1) = 3(x^2 + 4)$

Part(2)

The Cube Root of a Perfect Cube Number

- The cube root of a perfect cube number (a) is the number whose cube equals (a).

That is to find the cube root of a number means to find the number that, when multiplied by itself three times, gives that number.

- The symbol $\sqrt[3]{\quad}$ is used to denote the cube root of any number.



Note that

- $\sqrt[3]{0} = 0$

- The cube root of any number will have the same sign as that number.



Notes

① $\sqrt[3]{a^{3n}} = a^n$, where n is an integer.

For example : $\sqrt[3]{x^3} = x$

$\sqrt[3]{x^{12}} = x^4$

Example ⑤

Find each of the following in its simplest form :

1 $\sqrt[3]{125}$

2 $\sqrt[3]{-\frac{8}{343}}$

3 $\sqrt[3]{0.064}$

4 $\sqrt[3]{3\frac{3}{8}}$

5 $\sqrt[3]{(-7)^3}$

6 $\sqrt[3]{a^{15}}$

Example ⑥

Simplify to its simplest form : $\sqrt[3]{\frac{27}{64}} \times \sqrt{\frac{64}{9}} \times \left(\frac{9}{16}\right)^0$



Solution

$$\sqrt[3]{\frac{27}{64}} \times \sqrt{\frac{64}{9}} \times \left(\frac{9}{16}\right)^0 = \frac{1}{1} \frac{3}{4} \times \frac{8}{3} \times 1 = 2$$



Solve Equations Using the Cube Root

• If $X^3 = a$, then $X = \sqrt[3]{a}$

For example : If $X^3 = 125$, then $X = \sqrt[3]{125} = 5$

• If $\sqrt[3]{X} = a$, then $X = a^3$

For example : If $\sqrt[3]{X} = 4$, then $X = 4^3 = 64$

Example 7

Find the value of X in each of the following :

1 $\sqrt[3]{X} - 2 = 3$

2 $3X^3 + 15 = 96$

3 $2X^3 - 5 = -59$

Solution

1 $\therefore \sqrt[3]{X} - 2 = 3$

$\therefore \sqrt[3]{X} = 3 + 2 = 5$

$\therefore X = 5^3 = 125$

2 $\therefore 3X^3 + 15 = 96$

$\therefore 3X^3 = 96 - 15 = 81$

$\therefore X^3 = \frac{81}{3} = 27$

$\therefore X = \sqrt[3]{27} = 3$

3 $\therefore 2X^3 - 5 = -59$

$\therefore 2X^3 = -59 + 5 = -54$

$\therefore X^3 = \frac{-54}{2} = -27$

$\therefore X = \sqrt[3]{-27} = -3$

Example 8

A piece of wood in the shape of a cube with a volume of 729 cubic centimeters.
Calculate its lateral and total (surface) area.

Solution

Let the edge length of the cube be s ,

then its volume is s^3

$\therefore s^3 = 729$

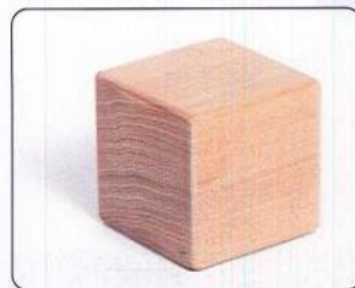
$\therefore s = \sqrt[3]{729} = 9$

Thus, the edge length of the cube is 9 cm.

The area of one face $= s^2 = 9^2 = 81$ square centimeters.

The lateral area $= 4s^2 = 4 \times 81 = 324$ square centimeters.

The total (surface) area $= 6s^2 = 6 \times 81 = 486$ square centimeters.



Exercises(2)

Complete each of the following :

1 $\sqrt[3]{8} + \sqrt[3]{-8} = \dots\dots\dots$

2 $\sqrt[3]{64} - \sqrt[3]{64} = \dots\dots\dots$

3 $\sqrt[3]{\dots\dots\dots} = 4$

4 $\sqrt[3]{\dots\dots\dots} = \sqrt[3]{27}$

5 $\sqrt[3]{\dots\dots\dots} = \sqrt{25}$

6 $\sqrt[3]{y^6} = \sqrt{\dots\dots\dots}$

7 $\sqrt{x^{12}} = \sqrt[3]{\dots\dots\dots}$

8 $\sqrt[3]{64 + \dots\dots\dots} = 5$

9 $\sqrt[3]{-27 a^6} = \dots\dots\dots$

10 If a cube has volume of 125 cubic centimeters , then its edge length = cm.

 Simplify each of the following :

1 $\sqrt{\frac{9}{4}} + \sqrt[3]{\frac{-27}{8}} + \left(\frac{4}{9}\right)^0$

2 $\left(\frac{3}{2}\right)^2 + \sqrt{\frac{25}{4}} + \sqrt[3]{\frac{125}{64}}$

3 $\sqrt[3]{\frac{125}{27}} \times \sqrt{\frac{81}{25}} \times \left(\frac{9}{5}\right)^0$

Find the value of x in each of the following :

1 $\sqrt[3]{x} = 5$

2 $\sqrt[3]{x} = -\sqrt{4}$

3 $x^3 = -125$

4  $x^3 + 5 = -22$

5 $2x^3 = 54$

6  $8x^3 - 15 = 49$

Find the solution set for each of the following equations in $\mathbb{Z}$:

1  $x^3 = 64$

2  $x^3 + 26 = -1$

3 $8x^3 + 27 = 0$

4  $3x^3 - 4 = 2x^3 + 4$

5 $(x+3)^3 = 343$

6  $(x-1)^3 + 2 = -6$



Homework

Choose the correct answer from the given ones :

1 If $\sqrt{x} = 5$, what is the value of x ?

- (a) 10 (b) 20
(c) 25 (d) ± 25

2 What is the value of $\sqrt{(-5)^2}$?

- (a) -5 (b) 5
(c) ± 5 (d) 25

3 Which of the following equals $\sqrt{16x^2}$?

- (a) $4x$ (b) $-4x$
(c) $4x^2$ (d) $4|x|$

4 What is the multiplicative inverse of the number $\sqrt{\frac{9}{25}}$ in simplest form?

- (a) $-\frac{3}{5}$ (b) $\frac{3}{5}$
(c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

5 What is the additive inverse of the number $-\sqrt{0.16}$ in simplest form?

- (a) -0.4 (b) 0.4
(c) -0.8 (d) 0.8

6 If a and b are the square roots of the number c , what is $a + b$ equal to?

- (a) $2a$ (b) $2b$
(c) 1 (d) 0

7 If $x = \sqrt{\frac{1}{9}}$, what is the value of x^3 ?

- (a) $\frac{1}{3}$ (b) $\frac{1}{9}$
(c) $\frac{1}{27}$ (d) $\frac{1}{81}$

8 $\sqrt{4 + \dots} = 4$

- (a) 0 (b) 4
(c) 12 (d) 16

9 If $a = -\frac{1}{2}$ and $b = -\frac{9}{8}$, then $\sqrt{ab} = \dots$

- (a) $\frac{9}{8}$ (b) $\frac{9}{16}$
(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

10 $\sqrt{36} + \sqrt{16} = \sqrt{\dots}$

- (a) 10 (b) 52
(c) 100 (d) 120

11 What is the value of $\sqrt[3]{0.008}$ in simplest form?

- (a) $\frac{1}{2}$ (b) 0.002
(c) $\frac{8}{10}$ (d) $\frac{1}{5}$

12 If $x^3 = -27$, what is the value of x ?

- (a) -3 (b) 3
(c) ± 3 (d) -9

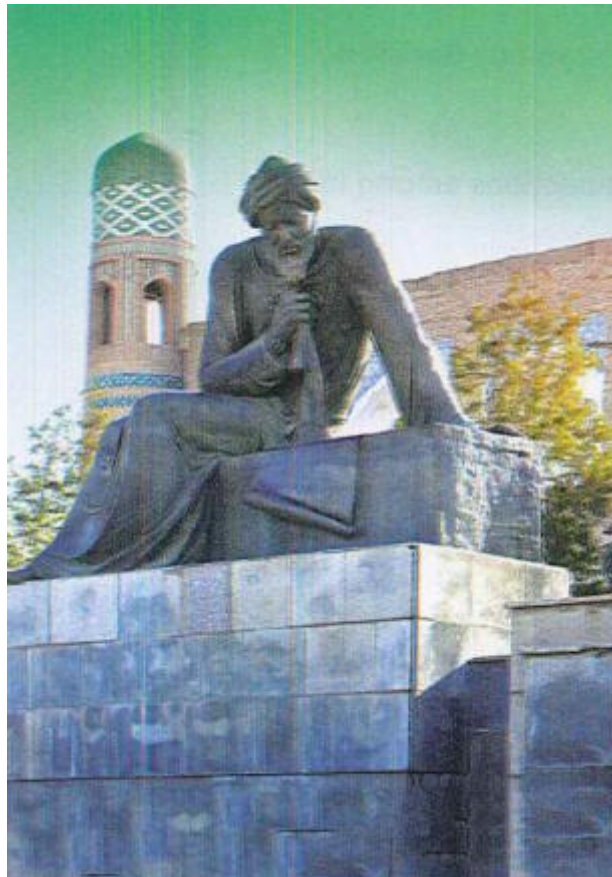
13 What is the value of $\sqrt[3]{64}$?

- (a) 2 (b) 4
(c) 8 (d) 64

14 If $\sqrt[3]{b} = -8$, what is the value of b ?

- (a) 2 (b) -2
(c) 64 (d) -512

Unit 2



Unit lessons:

Inequalities
Multiplying algebraic terms
Multiplying algebraic expression
Dividing algebraic expression by term
Dividing algebraic expressions

Lesson 1 : Inequalities

Concept of inequality

An inequality consists of two mathematical expressions, with one of the inequality signs ($>$, $<$, $\geq$, $\leq$) between them.

First degree inequality

Inequality contains only one variable raised to the exponent one

For example:

$$3x + 5 > 2 \quad , \quad 4x - 6 > 2(x - 1)$$

Writing the inequality

Express each of the following situations using an appropriate inequality:

- 1 The weight of a suitcase must be less than 18 kilograms.
- 2 A child must be at least 4 years old to be accepted into school.
- 3 A number, when 5 is subtracted from its four times, gives a result less than 12.
- 4 A number, when 3 is added to its double, gives a result greater than or equal to 7.

Solution

1 Let the mass of the suitcase be m kilograms

$$\therefore m < 18$$

2 Let the age of the child be X years

$$\therefore X \geq 4$$

3 Let the number be y , then its four times is $4y$

$$\therefore 4y - 5 < 12$$

4 Let the number be a , then its double is $2a$

$$\therefore 2a + 3 \geq 7$$

Properties of inequalities

If A , B and C are three numbers, then these have the following:

- 1) If $A > B$, then $A + C > B + C$.
- 2) If $A > B$, then $A - C > B - C$.
- 3) If $A > B$, $C > 0$ then $A \times C > B \times C$. & if $C < 0$ then $A \times C < B \times C$.
- 4) If $A > B$, $C > 0$ then $\frac{A}{C} > \frac{B}{C}$, & if $C < 0$ then $\frac{A}{C} < \frac{B}{C}$

Solving the Inequality

Means to find the values of the variable that satisfy the inequality,

Substitution set contains all possible values of the variable.

Solution set is a subset of substitution set, and its elements satisfy the inequality.

Ex (1):

Find the S.S in $\mathbb{Z}$:

(1) $4 - 2X \leq 2$

(2) $3(5X - 6) > 12$

Solutions:

(1) $4 - 2X \leq 2$

$-2X \leq 2 - 4$

$-2X \leq -2 \quad \div (-2)$

$X \geq 1$, then S.S = $\{1, 2, 3, 4, \dots\}$

(2) $3(5X - 6) > 12$

$15X - 18 > 12$

$15X > 12 + 18$

$15X > 30 \quad \div (15)$

$X > 2$, then S.S = $\{3, 4, 5, \dots\}$

Ex (2):

Find the solution set for the inequality: $\frac{3}{2}x - 1 \geq \frac{-2}{3}$

If the substitution set is :

1 $\mathbb{Q}$

2 $\mathbb{Z}$

 **Solution**

$$\therefore \frac{3}{2}x - 1 \geq \frac{-2}{3} \quad +1$$

$$\therefore x \geq \frac{1}{3} \div \frac{3}{2}$$

1 The solution set in $\mathbb{Q} = \{x : x \in \mathbb{Q}, x \geq \frac{2}{9}\}$

2 The solution set in $\mathbb{Z} = \{1, 2, 3, 4, \dots\}$

$$\therefore \frac{3}{2}x \geq \frac{-2}{3} + 1$$

$$\therefore x \geq \frac{1}{3} \times \frac{2}{3}$$

$$\therefore \left(\frac{3}{2}\right)x \geq \frac{1}{3} \quad \div \frac{3}{2}$$

$$\therefore x \geq \frac{2}{9}$$

Ex (3):

Find the solution set for the inequality: $5x + 3 \geq 18$ If the substitution set is : 1 $\mathbb{N}$ 2 $\mathbb{Z}$ 3 $\mathbb{Q}$ Find the solution set in $\mathbb{Z}$ for each of the following inequalities:

1 $7 - 5x \leq -23$

2 $3x - 1 \geq 2x + 1$

3 $4(2x - 5) > -4$

4 $3(2x - 4) < 2(x - 2)$

Ex (3):

Ahmed and three friends wish to purchase 4 meals of the same type, ensuring that the total bill does not exceed 700 LE, and knowing that the delivery charge is 20 LE.

Write an inequality that expresses the price of one meal, and solve the inequality to determine the maximum price for one meal.

**Solution:**

Let the price of one meal be x LE.

$$\therefore 4x + 20 \leq 700$$

$$\therefore 4x \leq 700 - 20$$

$$\therefore 4x \leq 680$$

$$\therefore x \leq \frac{680}{4}$$

$$\therefore x \leq 170$$

Thus, the maximum price for one meal is 170 LE.

Exercises

1 Express each of the following situations with an appropriate inequality :

- 1 To obtain the discount, your purchases must exceed 500 pounds.
- 2 An accountant with at least 3 years of experience is required.
- 3 The mass of the suitcase must not exceed 7 kilograms in order to carry it inside the aircraft cabin.
- 4 Your height must exceed 110 cm to play one of the games at an amusement park.
- 5 The maximum speed of your car is 80 km/h.
- 6 You should be at least 12 years old to use a mobile phone.

2 Find the solution set for each of the following inequalities if the substitution set is :

(a) $\mathbb{N}$

1 $x - 4 < 1$

4 $2x - 5 > -7$

(b) $\mathbb{Z}$

2 $2x \geq 10$

5 $2x + 5 \leq 11$

(c) $\mathbb{Q}$

3 $x + 2 > -1$

6 $4x - 1 < 11$

3 Find the solution set for each of the following inequalities in $\mathbb{N}$:

1  $x - 2 > 1$

2  $-2x \leq 0$

3  $2x - 3 < 7$

4 $\frac{2}{3}x \geq 1$

5  $\frac{1}{3}x + 3 \leq 1$

6 $\frac{1}{2}x + 4 \geq -2$

4 Find the solution set for each of the following inequalities in $\mathbb{Z}$:

1  $1 - 2x < 5$

2  $5 - 3x \geq 14$

3  $7 - 3x > -5$

4 $2 - 3x \leq 4$

5  $2(3 - 2x) < 4$

6  $3(2x - 1) > 9$

5 Find the solution set for each of the following inequalities in $\mathbb{Z}$:

1  $3x + 7 < 7x + 3$

2 $6y + 1 \leq 5y - 3$

3 $6x + 2 \geq 14 + 5x$

4 $3(x + 2) < -x + 4$

5  $2(2x + 3) \leq 5x + 2$

6  $4(x + 3) > 7x - 9$

6 Find the solution set for each of the following inequalities in $\mathbb{Q}$:

1  $2(x + 5) - 3 < 12$

2  $x - 2 \leq 3x + 7$

3 $8 - 2x \leq 5x$

4 $8x - 3x + 1 \leq 29$

5  $3(x - 7) \geq 7(x - 3)$

6 $2 - 3(x - 5) \geq x + 7$

- 10**  The maximum number of people an elevator can carry is 4, such that their total mass does not exceed 300 kg. If there are 3 people in the elevator with a total mass of 225 kg, write an inequality that expresses the mass x kg of the fourth person who can enter the elevator without violating the safety guidelines, and solve the inequality to find the maximum value of x .

- 11**  **Savings:** Hamza needs to save at least 250 pounds to buy a new toy. If he already has 100 pounds and he can save 20 pounds each week from his allowance, write an appropriate inequality to express this situation and solve it, then find the minimum number of weeks Hamza will need to save the money to buy the toy.

Homework

Choose the correct answer from the given ones :

1 What is the inequality that expresses that the temperature X is less than 40° ?

- (a) $X < 40^\circ$ (b) $X > 40^\circ$
(c) $X \leq 40^\circ$ (d) $X \geq 40^\circ$

2 What is the inequality that expresses that twice the number X is less than 5 ?

- (a) $X + 2 < 5$ (b) $X - 2 < 5$
(c) $2X < 5$ (d) $2X > 5$

3 If $-X < 5$, then which of the following is correct ?

- (a) $X > 5$ (b) $X > -5$
(c) $X < 5$ (d) $X < -5$

4 If $X \in \mathbb{N}$, then what is the solution set for the inequality $-X > 3$?

- (a) $\{4, 5, 6, \dots\}$ (b) $\{-4, -5, \dots\}$
(c) $\{-3\}$ (d) $\emptyset$

5 Which of the following inequalities expresses the following situation: "Omar needs at least two hours to complete the homework" ?

- (a) $X < 2$ (b) $X \leq 2$
(c) $X > 2$ (d) $X \geq 2$

6 If $X - 1 > 4$, which of the following could be the value of X ?

- (a) 3 (b) 4
(c) 5 (d) 7

7 Which of the following inequalities is equivalent to the inequality $\frac{X}{3} > 4$?

- (a) $X > \frac{4}{3}$ (b) $X < \frac{4}{3}$
(c) $X > 12$ (d) $X < 12$

8 Which of the following inequalities has $X = -7$ as one of its solutions in $\mathbb{Z}$?

- (a) $X > -7$ (b) $X < -7$
(c) $X > -6$ (d) $-X \geq -7$

9 Which of the following inequalities has $X = -4$ as one of its solutions in $\mathbb{Q}$?

- (a) $X - 2 \geq -4$ (b) $2X > -8$
(c) $X + 2 > -3$ (d) $-X > 4$

10 If $X > y$, then $\frac{1}{X} \dots\dots\dots \frac{1}{y}$
Where $X \neq 0, y \neq 0$

- (a) $<$ (b) $>$
(c) $=$ (d) $\geq$

Lesson 2 : Multiplying an algebraic term by an algebraic term or algebraic expression

When multiplying algebraic terms ,
follow these steps :

- ① Multiply the coefficients , apply the rules of signs.
- ② Multiply the variables , observe the addition of the exponents of the variables with the same base.

$$a x^m \times b x^n = a \times b x^{m+n}$$

Remember the rules of signs in multiplication

- The product of two numbers having the same sign is a positive number.
- The product of two numbers having different signs is a negative number.

For example :

$$5 x^2 \times 3 x^1 = (5 \times 3) x^{2+1} = 15 x^3$$

Add the exponents

Multiply the coefficients

Ex (1):

Find the product of each of the following :

1 $(5 a^3) (-2 a^5)$

2 $(-7 a^2 b^3) (2 a^4)$

Solution

1 $(5 a^3) (-2 a^5) = (5 \times -2) a^{3+5} = -10 a^8$

2 $(-7 a^2 b^3) (2 a^4) = (-7 \times 2) a^{2+4} b^3 = -14 a^6 b^3$

Ex (2):

Find the product of each of the following :

1 $(-7 y) (5 y^2)$

2 $(-3 a^2 b) (2 a^3)$

3 $(l m^2) (-3 l m^2 n) (5 l^2 m n^2)$

Multiplying algebraic term by algebraic expression

To find the product, use distribution property:

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

For example : $3x(x - 2) = (3x)(x) - (3x)(2) = 3x^2 - 6x$

Ex (3):

Find the result of each of the following :

1 $2m(3m - 5)$

2 $-3x(7x^2 - 5x + 2)$

3 $4ab(3a - 5b^2 + 3ab)$

 **Solution**

1 $2m(3m - 5) = (2m)(3m) - (2m)(5) = 6m^2 - 10m$

2 $-3x(7x^2 - 5x + 2) = (-3x)(7x^2) - (-3x)(5x) + (-3x)(2)$
 $= -21x^3 - (-15x^2) + (-6x)$
 $= -21x^3 + 15x^2 - 6x$

3 $4ab(3a - 5b^2 + 3ab) = (4ab)(3a) - (4ab)(5b^2) + (4ab)(3ab)$
 $= 12a^2b - 20ab^3 + 12a^2b^2$

Ex (4):

Find the result of each of the following :

1 $-7x(2x^2 - 5x)$

2 $-y^2(2xy + 5y - 9)$

3 $2l^2m^2(7lm^2 - 5m^3 + 4l)$

4 $\frac{2}{3}a(9a - 6b + 12)$

Ex (5):

Simplify the expression to the simplest form : $2x(3x-2) + 3x(x+1)$, then find the numerical value of the result when $x = 2$

 Solution

$$\begin{aligned}
 2x(3x-2) + 3x(x+1) &= (2x)(3x) - (2x)(2) + (3x)(x) + (3x)(1) \text{ (Distributive Property)} \\
 &= 6x^2 - 4x + 3x^2 + 3x \text{ (Multiply the terms)} \\
 &= 9x^2 - x \text{ (Combine like terms)}
 \end{aligned}$$

To find the numerical value of the result, substitute x with 2 and then calculate the result, taking the order of operations into consideration as follows :

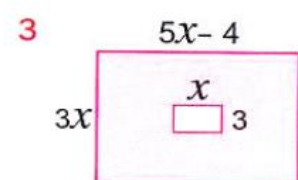
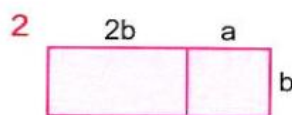
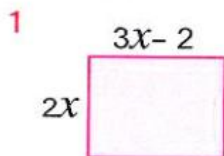
$$9 \times 2^2 - 2 = 9 \times 4 - 2 = 36 - 2 = 34$$

Ex (6):

Simplify the expression to the simplest form : $2a(a+4b) - 3b(a-3b)$, then find the numerical value of the result when $a = 1$, $b = -2$

Ex (7):

Find in the simplest form the algebraic expression that represents the area of the shaded part in each of the following :



Exercises

1 Complete each of the following :

1  $(-2a^2)(4a^5)$

2 $(-7y)(-8)$

3  $(-4a^3)(3a^5)$

4  $(-3a^3b^2)(-2ab^4)$

5  $(9x^3y)(-2x^2yz^5)$

6  $(5r^3s^2t)(-rt^4)$

2 Calculate the result of each of the following :

1 $a(a+1)$

2 $3x(7y-4z)$

3  $-4a(3a-2)$

4  $-3x(x-5)$

5 $-3(y+3)$

6  $-3a^2b(2ab^2-2b)$

7  $-4a(3a^2-2a+1)$

8  $2xy(4x^2+3xy^2-5y)$

9  $2x^2(4x^2-5x-7)$

10  $2x(4x^2-xy+5)$

3 Simplify to its simplest form :

1 $3a(a-b)+4a(2a+b)$

2 $3a(4a-2)-4a(3a-2)$

3  $3(5x^2+3x-2)-15x^2$

4  $4(3x^2+5x)-x(x^2-7x+8)$

5  $x(x^2-x-1)+3(2x^2+x+1)$

6  $2x(x^2-2x-3)-x^2(3x-5)$

7 $2x(x+y)-y(2x-y)+2(y^2-x^2)$

8 $3a(4a-1)+2a(a+3)-5a(2a-1)$

4 Simplify to the simplest form : $3a(4a-1)-a(5a+2)$, then find the numerical value of the result when $a=2$ 5 Simplify to the simplest form : $2x(3x-1)+3x(x+2)$, then find the numerical value of the resulting expression when $x=1$ 6 Simplify to the simplest form : $x(2x-y)-2y(x-y)$, then find the numerical value of the result when $x=2$, $y=-1$

7 Complete each of the following :

1 $6x^2y = 2x \times \dots\dots\dots$

2 $-4b^3c^4 = 2b^2c^2 \times \dots\dots\dots$

3 $x(\dots\dots\dots - 2x) = 6x - \dots\dots\dots$

4 $3x(\dots\dots\dots + 5x) = 6x^2 + \dots\dots\dots$

5 $2x(\dots\dots\dots - 5x) = 8x^3 - \dots\dots\dots$

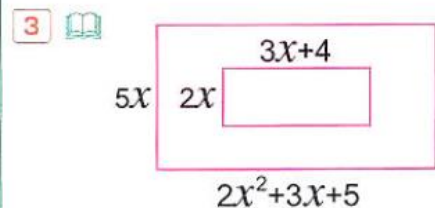
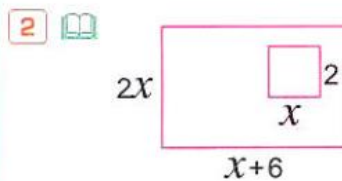
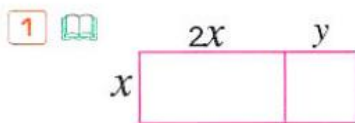
6 $3xy(\dots\dots\dots - \dots\dots\dots - 5x^2) = 6x^2y - 12xy^2 - \dots\dots\dots$

9 Solve each of the following equations in $\mathbb{Z}$:

1 $2x(x - 5) + 10x = 50$

2 $x(x - 2) + 2(x - 2) = 0$

3 $7x(x^2 + 2) = 2(x^3 + 7x - 20)$

10 Geometry : Find in the simplest form the algebraic expression that represents the area of the shaded part in each of the following :

Lesson Three

Multiplying Algebraic expressions

Multiplying a Binomial by Another Binomial

When multiplying a binomial by another binomial, multiply each term of the first binomial by each term of the second binomial using the distributive property.

$$(a + b)(c + d) = a(c + d) + b(c + d)$$

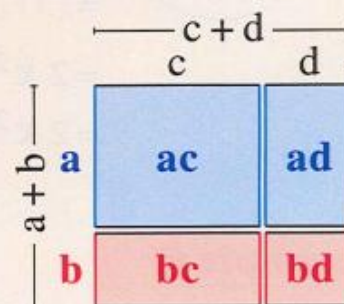
$$= ac + ad + bc + bd$$

$\uparrow$
 First
 $\times$
 First
 $\oplus$

$\uparrow$
 First
 $\times$
 Second
 $\oplus$

$\uparrow$
 Second
 $\times$
 First
 $\oplus$

$\uparrow$
 Second
 $\times$
 Second
 $\oplus$



For example :

$$(m + 4)(m + 3) = m(m + 3) + 4(m + 3)$$

$$= m^2 + 3m + 4m + 12 = m^2 + 7m + 12$$

Example :

Find the product : $(x + 5)(2x - 3)$

Solution

Using the horizontal method :

$$(x + 5)(2x - 3) = 2x^2 - 3x + 10x - 15$$

$$= 2x^2 + 7x - 15$$

Using the vertical method :

$$\begin{array}{r} 2x - 3 \\ \times x + 5 \\ \hline 2x^2 - 3x \\ 10x - 15 \\ \hline 2x^2 + 7x - 15 \end{array}$$

Multiplying by Inspection

In the previous example, multiplication can be done directly as follows :

$$(x + 5)(2x - 3) = \begin{array}{c} \text{First term} \quad \text{Second term} \\ \text{First} \times \text{First} \quad \oplus \quad \text{Product of means} + \text{Product of extremes} \quad \oplus \quad \text{Second} \times \text{Second} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ = (x)(2x) + ((5)(2x) + (x)(-3)) + (5(-3)) \\ = 2x^2 + (10x - 3x) + (-15) \\ = 2x^2 + 7x - 15 \end{array}$$



Example :

Find by inspection the product of each of the following :

1 $(2a + 3)(5a + 1)$

2 $(3x + 4)(2x - 5)$

Solution

$$1 \quad (2a + 3)(5a + 1) = 10a^2 + (15a + 2a) + 3$$

$$= 10a^2 + 17a + 3$$

$$2 \quad (3x + 4)(2x - 5) = 6x^2 + (8x - 15x) - 20$$

$$= 6x^2 - 7x - 20$$

Special Cases**1 Expansion of the square of a binomial**

$$1 \quad (a + b)^2 = (a + b)(a + b)$$

First term Second term

$$= a(a + b) + b(a + b)$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2$$

Square of the first term

Twice the product of the first term and the second term

Square of the second term

For example :

$$(x + 5)^2 = x^2 + 2 \times x \times 5 + 5^2$$

$$= x^2 + 10x + 25$$

$$2 \quad (a - b)^2 = (a - b)(a - b)$$

First term Second term

$$= a(a - b) - b(a - b)$$

$$= a^2 - ab - ba + b^2$$

$$= a^2 - 2ab + b^2$$

Square of the first term

Twice the product of the first term and the second term

Square of the second term

For example :

$$(x - 3)^2 = x^2 - 2 \times x \times 3 + 3^2$$

$$= x^2 - 6x + 9$$

Example :

Find the expansion of each of the following :

1 $(3a + 5)^2$

2 $(5a + 2b)^2$

Solution

$$1 \quad (3a + 5)^2 = (3a)^2 + 2 \times 3a \times 5 + (5)^2$$

$$= 9a^2 + 30a + 25$$

$$2 \quad (5a + 2b)^2 = (5a)^2 + 2 \times 5a \times 2b + (2b)^2$$

$$= 25a^2 + 20ab + 4b^2$$

2 The product of the sum of two terms and the difference between them

$$\begin{aligned}
 (a + b)(a - b) &= a(a - b) + b(a - b) \\
 &= a^2 - \cancel{ab} + \cancel{ba} - b^2 = a^2 - b^2
 \end{aligned}$$

First term Second term Square of the first term Square of the second term

For example : $(a + 3)(a - 3) = a^2 - 9$

Example :

Find in the simplest form each of the following :

1 $(2m - 5)(2m + 5)$

2 $(5x + 3y)(5x - 3y)$

3 $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right)$

4 $(7 - 2a)(7 + 2a) - 49$

Solution

1 $(2m - 5)(2m + 5) = (2m)^2 - 5^2 = 4m^2 - 25$

2 $(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2$

3 $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right) = \left(\frac{1}{3}a\right)^2 - \left(\frac{2}{5}b\right)^2 = \frac{1}{9}a^2 - \frac{4}{25}b^2$

4 $(7 - 2a)(7 + 2a) - 49 = (7)^2 - (2a)^2 - 49 = 49 - 4a^2 - 49 = -4a^2$

Multiplying a Binomial by an Algebraic Expression Consisting of More Than Two Terms

Example :

Find the product : $(x - 3)(4x^2 + 2x - 7)$

Solution

Using the horizontal method :

$$\begin{aligned}
 (x - 3)(4x^2 + 2x - 7) &= 4x^3 + 2x^2 - 7x - 12x^2 - 6x + 21 \\
 &= 4x^3 - 10x^2 - 13x + 21
 \end{aligned}$$

Using the vertical method :

$$\begin{array}{r}
 4x^2 + 2x - 7 \\
 \times x - 3 \\
 \hline
 4x^3 + 2x^2 - 7x \\
 -12x^2 - 6x + 21 \\
 \hline
 4x^3 - 10x^2 - 13x + 21
 \end{array}$$

Summary of multiplying algebraic expressions

Multiplying two algebraic expressions

$$(a + b)(c + d) = ac + ad + bc + bd$$

For example :

$$\begin{aligned}
 (m + 3)(n + 2) \\
 = mn + 2m + 3n + 6
 \end{aligned}$$

Multiplying by inspection

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

For example :

$$\begin{aligned}
 (x + 4)(x - 5) \\
 = x^2 + (4 - 5)x - 20 \\
 = x^2 - x - 20
 \end{aligned}$$

Expansion of the square of a binomial

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

For example :

$$\begin{aligned}
 (x - 3)^2 \\
 = x^2 - 2 \times x \times 3 + 3^2 \\
 = x^2 - 6x + 9
 \end{aligned}$$

The product of the sum of two terms and the difference between them

$$(a + b)(a - b) = a^2 - b^2$$

For example :

$$\begin{aligned}
 (x - 4)(x + 4) \\
 = x^2 - 4^2 \\
 = x^2 - 16
 \end{aligned}$$

Exercises

Complete each of the following :

1) $(x + 3)(x + 2) = \dots + 5x + 6$

2) $(x + 2)(x - 5) = x^2 + \dots - 10$

3) $(a - 3)(a - 7) = a^2 - \dots + \dots$

4) $(2x - 5)(x + 7) = \dots + \dots - 35$

5) $(4x - 3y)(2x + 5y) = 8x^2 + \dots - 15y^2$

6) $(x - 7)(x + 7) = x^2 - \dots$

7) $(2x - 3)^2 = 4x^2 - \dots + \dots$

8) $(x + 5y)^2 = \dots + \dots + 25y^2$

Find the product of each of the following :

1) $(a + 3)(a + 4)$

2) $(b^2 - 4)(b^2 + 2)$

3) $(x + 1)(x - 6)$

4) $(3x + 1)(x - 3)$

5) $(x - 7)(2x - 1)$

6) $(m + 4n)(2m - n)$

7) $(3x - 1)(2x + 5)$

Find the expansion of each of the following :

1) $(a + 3)^2$

2) $(x + 7)^2$

3) $(2y + 3)^2$

4) $(2x - 9)^2$

5) $(x - 6)^2$

6) $(x - 3y)^2$

Find the product of each of the following :

1  $(a - 4)(a + 4)$

2  $(4x - 3)(4x + 3)$

3 $(6x - 5y)(6x + 5y)$

4  $(3b + 2)(3b - 2)$

5 $(lm + 6n)(lm - 6n)$

6  $\left(\frac{1}{2}x + 1\right)\left(\frac{1}{2}x - 1\right)$

Find the simplest form of each of the following :

1  $(x - 2)(x^2 - 3x + 5)$

2  $(x + 2)(x^2 - x + 3)$

3  $(2x - 1)(x^2 - 3x + 4)$

4 $(2x - y)(2x^2 - 3xy + y^2)$

Find the simplest form of each of the following :

1 $(x - 3)^2 - 9$

2 $(x - 2)(x + 2) - x(x + 1)$

3 $(x + 1)^2 - x(x + 2)$

4 $(2x + 3)^2 + (x - 2)(x + 5)$

5  $(a + b)^2 - (a + b)(a - b)$

6  $(4a - 3b)^2 - (4a - 3b)(4a + 3b)$

Homework

Choose the correct answer from the given ones :

1 What is the number of terms in the expression resulting from the product of $(X - 3)(X + 4)$ in the simplest form ?

- (a) 1 (b) 2
(c) 3 (d) 4

2 If $(X - 5)(X + 2) = X^2 + bX + c$, what is the value of c ?

- (a) -10 (b) -7
(c) 7 (d) 10

3 If $(3X - 7)^2 = aX^2 + bX + c$, what is the value of b ?

- (a) -42 (b) -21
(c) 21 (d) 42

4 If $(X - 5)(X + 5) = X^2 + bX + c$, what is the value of b ?

- (a) -25 (b) -10
(c) 0 (d) 10

5 If $(X - 3)(X + 3) = X^2 - k$, what is the value of k ?

- (a) 9 (b) 6
(c) -9 (d) -6

6 What is the coefficient of a in the expansion of the expression $(4a - 5b)^2$?

- (a) 40 (b) 20
(c) -20 (d) -40

7 If $X + y = 15$, $X - y = 5$, what is the value of $X^2 - y^2$?

- (a) 75 (b) 20
(c) 10 (d) 3

8 If $(X + y)^2 = 16$, $XY = 3$, what is the value of $X^2 + y^2$?

- (a) $5\frac{1}{3}$ (b) 10
(c) 13 (d) 48

9 What is the result of subtracting $(a + b)^2$ from $(a - b)^2$?

- (a) $2ab$ (b) $-2ab$
(c) $-4ab$ (d) $4ab$

10 If $y^2 = 7$, and $X^2 = 10$, then what is the value of $(X + y)(X - y)$?

- (a) 70 (b) 17
(c) 3 (d) -3

11 If $X^2 + y^2 = 20$, and $(X + y)^2 = 26$, then what is the value of XY ?

- (a) 3 (b) 6
(c) 9 (d) 12

12 If $X^2 = 16$, $y^2 = 9$, and $XY = 12$, then what is the value of $(X - y)^2$?

- (a) 49 (b) 165
(c) -1 (d) 1

Lesson four

Dividing an Algebraic term or an Algebraic Expression by an Algebraic term

Division an Algebraic Term by another Algebraic Term

When dividing an algebraic term by another algebraic term , follow the steps :

- ① Divide the coefficients of the terms , apply the rule of signs.
- ② Divide the symbolic factors , subtract the exponents of the variables with the same bases (subtract the exponents of the divisor from the exponents of the dividend).

Remember the rule of signs in division

- The quotient of two numbers with the same sign is a positive number.
- The quotient of two numbers with different signs is a negative number.

For example :
$$\begin{array}{c} \text{Subtract the exponents} \\ (-12 a^5) \div (3 a^2) = \left(\frac{-12}{3} \right) a^{5-2} = -4 a^3 \\ \text{Divide the coefficients} \end{array}$$

Note that : Division by zero is undefined ; therefore , in all problems involving variables , the divisor must not equal zero.

Example :

Find the quotient of each of the following :

1 $\frac{12 a^3}{3 a}$

2 $21 x \div (-3)$

3 $\frac{-15 x^2 y^3}{5 x y^2}$

4 $\frac{-24 a^5 b^3 c^2}{-8 a^2 b^3}$

Solution

1 $\frac{12 a^3}{3 a} = 4 a^2$

2 $21 x \div (-3) = -7 x$

3 $\frac{-15 x^2 y^3}{5 x y^2} = -3 x y$

4 $\frac{-24 a^5 b^3 c^2}{-8 a^2 b^3} = 3 a^3 c^2$

Division an Algebraic Expression by an Algebraic Term

When dividing an algebraic expression by an algebraic term , divide each term of the expression by this term as follows :

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

For example : $\frac{6x^2 + 2xy}{2x} = \frac{6x^2}{2x} + \frac{2xy}{2x} = 3x + y$

Example:

Find the quotient of each of the following :

1 $(12x^2 - 40x^3 - 4x) \div (4x)$

2 $\frac{-21ab^2c - 14a^2bc + 35abc^2}{-7abc}$

3 $\frac{-6x(2x^2 - 3x^4 + 5)}{3x}$

Solution

$$\begin{aligned} 1 \quad (12x^2 - 40x^3 - 4x) \div (4x) &= \frac{12x^2}{4x} + \frac{-40x^3}{4x} + \frac{-4x}{4x} \\ &= 3x - 10x^2 - 1 \end{aligned}$$

$$\begin{aligned} 2 \quad \frac{-21ab^2c - 14a^2bc + 35abc^2}{-7abc} &= \frac{-21ab^2c}{-7abc} + \frac{-14a^2bc}{-7abc} + \frac{35abc^2}{-7abc} \\ &= 3b + 2a - 5c \end{aligned}$$

$$\begin{aligned} 3 \quad \frac{-6x(2x^2 - 3x^4 + 5)}{3x} &= \frac{-12x^3 + 18x^5 - 30x}{3x} = \frac{-12x^3}{3x} + \frac{18x^5}{3x} + \frac{-30x}{3x} \\ &= -4x^2 + 6x^4 - 10 \end{aligned}$$

Exercises

Find the quotient of each of the following :

1 $\frac{6a}{2}$

4 $\frac{-45a^5}{-3a^3}$

7 $\frac{-12x^5y^2}{-4x^2y}$

2 $\frac{12x}{-x}$

5 $\frac{18x^2y^3}{-2x^2y}$

8 $\frac{15a^3b^2c}{3a^2b^2c}$

3 $(27x^3) \div (9x)$

6 $\frac{9x^4}{-3x^3}$

9 $-18x^2 \div (-3x^2)$

Find the quotient of each of the following :

1 $\frac{26a^2 + 14a^4}{2a}$

3 $\frac{18x^3 + 12x^2 - 6x}{-6x}$

5 $\frac{32x^5 - 48x^3 + 72x^7}{-8x^3}$

7 $\frac{-15a^3x^2 + 10a^4x^3}{-5a^3x^2}$

9 $(2x - 4x^2 + 8x^3) \div (2x)$

2 $\frac{18m^4 + 32m^2}{-2m^2}$

4 $\frac{3ab^2 + 9a^2b - 6a^2b^2}{3ab}$

6 $\frac{15x^3y^2 + 6xy^2 - 3xy}{-9xy}$

8 $\frac{49x^3 - 14x^2 + 21x}{-7x}$

10 $(x - x^2 - x^3) \div (-x)$

Find the simplest form of each of the following :

$$1) \frac{x^2}{-x} + \frac{-4x}{x} - \frac{3x^3}{x^2}$$

$$2) \frac{28x^2 - 42x}{7x} + \frac{14x^2 - 35x}{-7x}$$

$$3) \frac{2x(6x^2 - 2x + 8)}{4x}$$

$$4) \frac{48x^4 - 144x^3 - 96x^2}{-6x \times 8x}$$

Complete each of the following:

$$1) -6x^3y \div 2xy = \dots\dots\dots$$

$$2) \frac{-24x^4}{\dots\dots\dots} = -6x$$

$$3) \dots\dots\dots \times 24x^2y = 72x^2y^2$$

$$4) \frac{4y^5}{y^3} + 2y^2 = \dots\dots\dots$$

If the area of a rectangle equals $(4x^4 + 8x^3 + 12x^2)$ square units , and one of its dimensions is $4x^2$ length units , find the other dimensions in terms of x

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Homework

Choose the correct answer from the given ones :

1 $-12x^3 \div (-4x) = \dots\dots\dots$

- (a) $3x^2$ (b) $-3x^2$
(c) $-3x$ (d) $48x^4$

2 $24x^3 \div (-6x^2) = \dots\dots\dots$

- (a) -4 (b) $-4x$
(c) $-4x^5$ (d) $-4x^2$

3 If $\frac{8x^2}{a} = 1$, what is the value of a?

- (a) -1 (b) 1
(c) $-8x^2$ (d) $8x^2$

4 $\frac{a+b}{c} = \dots\dots\dots$

- (a) $a + \frac{b}{c}$ (b) $\frac{a}{c} + b$
(c) $\frac{a}{c} + \frac{b}{c}$ (d) $\frac{ab}{c}$

5 $(x^2 \div x) + x = \dots\dots\dots$

- (a) Zero (b) $2x$
(c) $2x + 1$ (d) $x + 1$

6 $(x^3 + x^2) \div x^2 = \dots\dots\dots$

- (a) 0 (b) x
(c) $x + 1$ (d) $2x + 1$

7 $\frac{3x^2 - 6x}{3x} = \dots\dots\dots$

- (a) $-x$ (b) $-x^2$
(c) $x^2 - 2x$ (d) $x - 2$

8 $\dots\dots\dots \div (-2x^2y) = 12xy^2$

- (a) $6xy$ (b) $-6xy$
(c) $24x^3y^3$ (d) $-24x^3y^3$

9 $\frac{5x^2y - \dots\dots\dots}{5xy} = x - 3y$

- (a) $\frac{3}{5}xy$ (b) $\frac{3}{5}yx^2$
(c) $15x^2y$ (d) $15xy^2$

10 $\frac{21ab^2 - 28a^2b^2}{\dots\dots\dots} = 3b - 4ab$

- (a) $3b$ (b) $7ab^2$
(c) $7a^2b^2$ (d) $7ab$

11 $10ab^2 \div \dots\dots\dots = -2ab$

- (a) $5b$ (b) $-5b$
(c) $-12ab$ (d) $-5b^2$

12 If $(4x^2 + kxy) \div (4x) = x - 2y$, then what is the value of $|k|$?

- (a) 2 (b) 4
(c) 6 (d) 8

Lesson 5 Dividing Algebraic Expressions

To divide expression by another expression, follow the following steps:

- ① Arrange each expression in a descending order.
- ② Divide first term of dividend by first term of divisor.
- ③ Multiply the result by the terms of divisor and subtract result from the dividend.
- ④ Repeat these steps to the end.

Example (1):

Divide $(x^2 + x - 12)$ by $(x + 4)$

divisor quotient
dividend

Solution:

To perform the division, follow these steps :

- 1 Arrange the terms of both the dividend and the divisor in descending (or ascending) order according to the exponents of one of the variables (used descending order is preferred), and write the like terms one below the other.
- 2 Divide x^2 by x to get x .
- 3 Multiply x by $(x + 4)$ to obtain $x^2 + 4x$.
- 4 Subtract $(x^2 + 4x)$ from $(x^2 + x - 12)$ to obtain $-3x - 12$.
- 5 Repeat steps 2, 3, and 4 in order until the remainder equals zero, at which point the division process is complete, and the quotient is $x - 3$.

$$\begin{array}{r}
 x - 3 \\
 x + 4 \overline{) x^2 + x - 12} \\
 \underline{-(x^2 + 4x)} \\
 -3x - 12 \\
 \underline{+ (3x + 12)} \\
 0
 \end{array}$$

Example 2

Find the quotient of $(12 - 5x^2 + 6x^3 - 14x)$ divided by $(2x - 3)$

Solution

Arrange the terms of the dividend and the divisor in descending order according to the exponent of x before performing the division.

$$\begin{array}{r}
 3x^2 + 2x - 4 \\
 2x - 3 \overline{) 6x^3 - 5x^2 - 14x + 12} \\
 \underline{-(6x^3 - 9x^2)} \\
 4x^2 - 14x + 12 \\
 \underline{-(4x^2 - 6x)} \\
 -8x + 12 \\
 \underline{+ (8x - 12)} \\
 0
 \end{array}$$

Thus, the quotient is $3x^2 + 2x - 4$

Example ③

Find the quotient of $(X^3 + X + 10)$ divided by $(X + 2)$

Solution

$$\begin{array}{r}
 \overline{X^2 - 2X + 5} \\
 X+2 \overline{X^3 + X + 10} \\
 \underline{-X^3 + 2X^2} \\
 -2X^2 + X + 10 \\
 \underline{+2X^2 - 4X} \\
 5X + 10 \\
 \underline{-5X } \\
 0
 \end{array}$$

Thus : the quotient is $X^2 - 2X + 5$

**Notes that**

The dividend does not include X^2 , so a blank space is left when performing the division.

Example ⑥

A rectangular piece of land has an area of $(8X^2 + 6X - 9)$ square meters. If the length of the piece of land is $(4X - 3)$ meters, find its width, then calculate the perimeter when $X = 5$

Solution

$$\therefore \text{The width of the piece of land} = \frac{\text{the area}}{\text{the length}} = \frac{8X^2 + 6X - 9}{4X - 3}$$

$$\therefore \text{The width of the piece of land} = (2X + 3) \text{ meters.}$$

When $X = 5$:

$$\therefore \text{The width} = 2 \times 5 + 3 = 10 + 3 = 13 \text{ m.}$$

$$\therefore \text{The length} = 4 \times 5 - 3 = 20 - 3 = 17 \text{ m.}$$

$$\therefore \text{The perimeter} = 2(17 + 13) = 2 \times 30 = 60 \text{ m.}$$

$$\begin{array}{r}
 \overline{2X + 3} \\
 4X-3 \overline{8X^2 + 6X - 9} \\
 \underline{-8X^2 + 6X} \\
 12X - 9 \\
 \underline{-12X } \\
 0
 \end{array}$$

TRY it Yourself 3

Find the quotient of each of the following:

1 $X^2 - 64$ divided by $X - 8$

2 $X^3 + 27$ divided by $X + 3$

Exercises

1 Find the quotient of each of the following expressions:

1 $x^2 + 5x + 6$ divided by $x + 2$

2 $x^2 + 9x + 20$ divided by $x + 4$

3 $x^2 - 10x + 25$ divided by $x - 5$

4 $x^2 - 4x - 12$ divided by $x - 6$

2 Find the quotient of each of the following expressions :

1 $x^2 + 6 - 5x$ divided by $x - 3$

2 $x + x^2 - 2$ divided by $x - 1$

3 $2 + 2y^2 - 5y$ divided by $y - 2$

4 $x^3 + 4x^2 - 5$ divided by $x - 1$

5 $x^2 - 1$ divided by $x + 1$

6 $x^3 - 27$ divided by $x - 3$

3 Find the quotient of each of the following expressions:

1 $x^3 + 5x^2 + 7x + 2$ divided by $x + 2$

2 $7x - 5x^2 + 2x^3 - 6$ divided by $2x - 3$

3 $15 - 7x^2 + 3x - 4x^3$ divided by $5 - 4x$

5 Divide $(-3x^2 + x^3 - x + 6)$ by $(x - 2)$, then find the numerical value of the quotient when $x = 3$ **6 Find the other factor of each of the following :**

1 If $(x - 4)$ is a factor of the expression $(x^2 - 5x + 4)$

2 If $(2x + 1)$ is one factor of the expression $(2x^2 - 7x - 4)$

8 Find the value of b that makes the expression $(4x^2 + 11x + b)$ divisible by $(4x - 1)$

9 If the expression $(x^3 - x^2 - 4x - m)$ is divisible by $(x - 3)$, find the value of m .

13 A theater sells tickets with total revenues of $(4x^2 + 16x + 12)$ pounds. If the price of each ticket is $(4x + 4)$ pounds, determine the number of tickets sold in terms of x



Homework

Choose the correct answer from the given ones :

1 What is the result of dividing
 $(x^2 - 12x + 20)$ by $(x - 2)$?

- (a) $x - 4$ (b) $x + 4$
 (c) $x - 10$ (d) x

2 What is the result of dividing
 $(2x^2 - 7x + 5)$ by $(2x - 5)$?

- (a) $x - 1$ (b) $x + 1$
 (c) $x + 5$ (d) x

3 If $\frac{2x + a}{x + 3} = 2$, what is the value of a ?

- (a) 2 (b) 3
 (c) 5 (d) 6

4 If $\frac{3x + 15}{x - a} = 3$, what is the value of a ?

- (a) -5 (b) -3
 (c) 3 (d) 5

5 If $\frac{x - 2}{2 - x} = a$, what is the value of a ?

- (a) -2 (b) -1
 (c) 1 (d) 2

6 If the quotient of $(x^2 - 2x - 35)$ divided by $(x + 5)$ is $(x + b)$, what is the value of b ?

- (a) -7 (b) -5
 (c) 5 (d) 7

7 What is the value of m that makes the expression $(x^2 - 5x + m)$ divisible by $(x - 2)$?

- (a) 6 (b) 2
 (c) 5 (d) 1

8 If the quotient of $(x^3 - 4x)$ divided by $(x - 2)$ is $(ax + x^2)$, what is the value of a ?

- (a) -4 (b) -2
 (c) 2 (d) 4

9 If the area of a rectangle is $(x^2 + 12x + 35)$ square meters, and one of its dimensions is $(x + 5)$ meters, what is the other dimension?

- (a) $(x - 7)$ meters (b) $(x + 7)$ meters
 (c) $(x - 5)$ meters (d) $(7 - x)$ meters

10 If the area of a parallelogram is $(2x^2 + 7xy + 5y^2)$ square units, and the length of its base is $(x + y)$ units, what is the corresponding height for this base in length units?

- (a) $2x - 5y$ (b) $2x + 7y$
 (c) $5y + 2x$ (d) $7y - 2x$



Homework

Choose the correct answer from the given ones :

1  $(2x)(3x) = \dots\dots\dots$

- (a) $5x$ (b) $6x$
(c) $5x^2$ (d) $6x^2$

2  $(-3x^2)(4x^3) = \dots\dots\dots$

- (a) $-12x^5$ (b) $12x$
(c) $-12x^6$ (d) $12x^5$

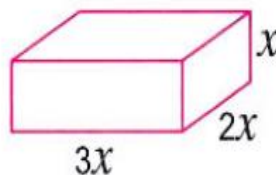
3 $(3a^4b)(5a^2b^2)(2a^3) = \dots\dots\dots$

- (a) $60a^{11}b^3$ (b) $30a^{10}b^2$
(c) $15a^{10}b^3$ (d) $30a^9b^3$

4 If the edge length of a cube is $2b$, then what is the volume of this cube ?

- (a) $4b^2$ (b) $2b^3$
(c) $4b^3$ (d) $8b^3$

5 What is the volume of the given rectangular prism ?



- (a) $6x^3$ (b) $6x$
(c) $5x^3$ (d) $6x^2$

6  $2(x+3) = \dots\dots\dots$

- (a) $2x^2 + 6x$ (b) $2x + 3$
(c) $2x + 6$ (d) $x + 6$

7  $x(x-1) + x = \dots\dots\dots$

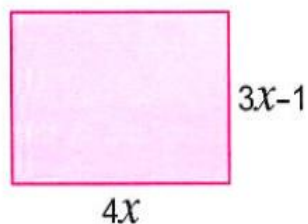
- (a) $x(2x-1)$ (b) $2x^2$
(c) x^2 (d) $x^2 - x$

8 If $a(3x-2) = 12x^2 - 8x$, then what is the value of a ?

- (a) $3x$ (b) 4
(c) $4x$ (d) $4x^2$

9 What is the area of the given rectangle?

- (a) $12x - 1$
(b) $12x^2 - 4x$
(c) $12x + 4x$
(d) $(12x)(4x)$

10 If $a + 3b = 7$, and $c = 3$, then what is the numerical value of the expression : $a + 3(b+c)$?

- (a) 10 (b) 13
(c) 15 (d) 16

Unit 3

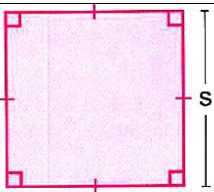
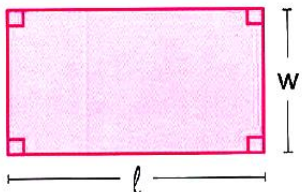
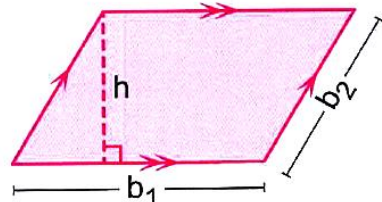
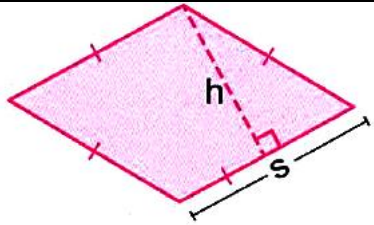


Unit lessons:

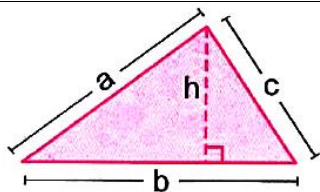
Areas
Geometrical constructions
Geometrical transformations

Lesson 1-U3: Area and perimeter

We have studied before some rules about Areas and perimeters of some shapes:

Shape	Example
Square  $P = s \times 4$ $A = s^2$	If the side length of a square is 3 cm, then: - The perimeter of the square : $P = s \times 4 = 3 \times 4 = 12 \text{ cm.}$ - The area of the square : $A = s^2 = 3^2 = 9 \text{ cm}^2.$
Rectangle  $P = 2 (\ell + w)$ $A = \ell \times w$	If the dimensions of the rectangle are 8 cm and 5 cm, then: - The perimeter of the rectangle : $P = 2 (\ell + w) = 2 (8 + 5) = 2 \times 13 = 26 \text{ cm.}$ - The area of the rectangle : $A = \ell \times w = 8 \times 5 = 40 \text{ cm}^2.$
Parallelogram  $P = 2 (b_1 + b_2)$ $A = b_1 \times h$	If the base lengths of the parallelogram are 4 cm and 6 cm and its smaller height is 3 cm, then: - The perimeter of the parallelogram : $P = 2 (b_1 + b_2) = 2 (6 + 4) = 2 \times 10 = 20 \text{ cm}$ - The area of the parallelogram : $A = b \times h = 6 \times 3 = 18 \text{ cm}^2.$
Rhombus  $P = 4s$ $A = s \times h$	If the side length of a rhombus is 7 cm and its height is 5 cm, then: - The perimeter of the rhombus : $P = 4s = 4 \times 7 = 28 \text{ cm.}$ - The area of the rhombus : $A = s \times h = 7 \times 5 = 35 \text{ cm}^2.$

Triangle



$$P = a + b + c$$

$$A = \frac{1}{2} b \times h$$

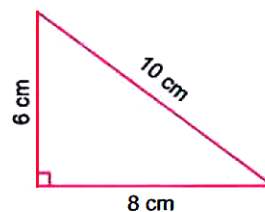
If the side lengths of the triangle are as shown in the figure, then:

- The perimeter of the triangle :

$$P = a + b + c \\ = 6 + 8 + 10 = 24 \text{ cm.}$$

- The area of the triangle :

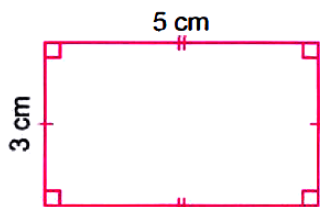
$$A = \frac{1}{2} b \times h = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2.$$



Practicese

Find the perimeter and area of each of the following shapes:

1



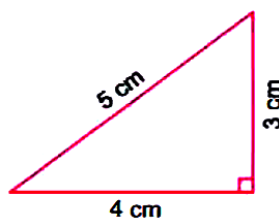
Perimeter =

=

Area =

=

2



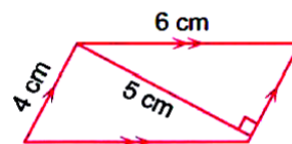
Perimeter =

=

Area =

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3



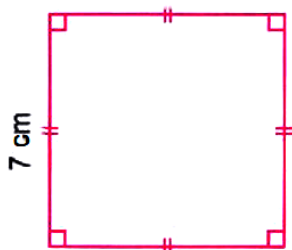
Perimeter =

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Area =

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4



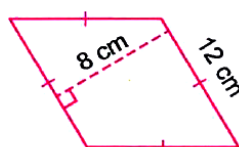
Perimeter =

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Area =

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5



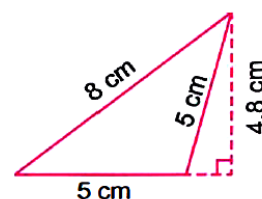
Perimeter =

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Area =

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6



Perimeter =

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Area =

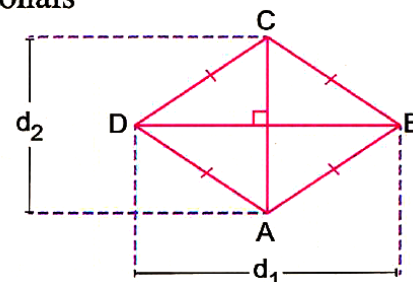
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Area of The Rhombus Given The Lengths of Its Diagonals

Area of the rhombus = $\frac{1}{2}$ the product of the lengths of its diagonals

That is: $A = \frac{1}{2} \times d_1 \times d_2$

"where d_1 and d_2 are the lengths of the diagonals of the rhombus"



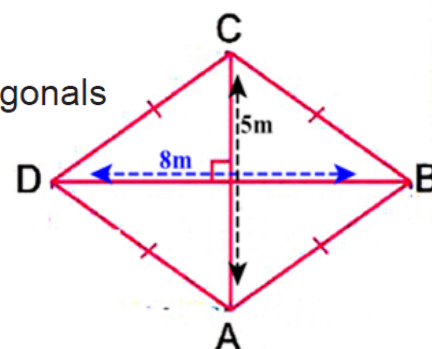
Example

For a rhombus with diagonal lengths of 5 meters and 8 meters, find its area.

Area of the rhombus = $\frac{1}{2}$ Product of the lengths of its diagonals

$$A = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 5 \times 8 = 20$$

Thus, the area of the rhombus = 20 square meters



Practicese

- 1 A rhombus with diagonal lengths of 4 meters and 6 meters. Find its area.

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- 2 A piece of land in the shape of a rhombus with diagonal lengths of 6 yards and 5 yards. Find its area.

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- 3 A rhombus with an area of 24 square inches and the length of one of its diagonals is 4 inches. Find the length of the other diagonal.

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- 4 A rhombus with a side length of 10 feet and the length of its diagonals 12 feet and 16 feet. Find its height.

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Area of the Square Given the Length of its Diagonal

A square is a rhombus with diagonals equal in length; thus:

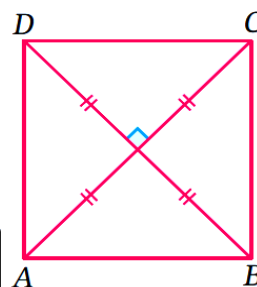
The area of the square = $\frac{1}{2} \times \text{length of the diagonal}$
 $\times \text{length of the diagonal}$

hence,

the area of the square = $\frac{1}{2}$ square of the length of its diagonal

Assuming the area of the square A , and the length of the diagonal d ,

So: $A = \frac{1}{2} d^2$



Example the square with a diagonal length of 6 cm has an area in square centimeters:

$$A = \frac{1}{2} \times 6^2 = \frac{1}{2} \times 36 = 18$$

Practicese

- 1** A square with a diagonal length of 8 yards. Find its area.

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- 2** A square with a diagonal length of 7 cm. Find its area.

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- 3** A square with an area of 8 square meters. Find the length of its diagonal.

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Which has a greater area ?

A square with a diagonal length of 16 cm or a rectangle with dimensions of 12 cm and 11 cm.

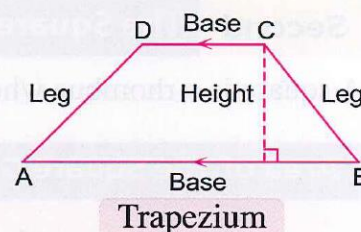
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The Trapezium

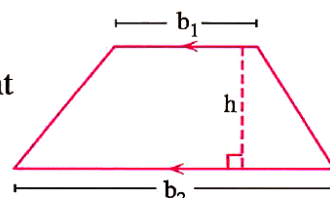
A trapezium is a quadrilateral in which only two sides are parallel, and each of the parallel sides is called a "**base**", while each of the non-parallel sides is called a "**leg**".



Area of The Trapezium

The area of the trapezium
 $= \frac{1}{2} \times \text{the sum of the lengths of the two parallel bases} \times \text{the height}$

That is: $A = \frac{1}{2} (b_1 + b_2) \times h$

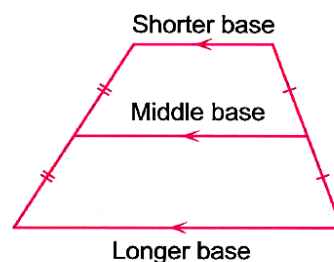


"Where b_1 and b_2 are the lengths of the two parallel bases, and h is the height".

The middle base of the trapezium

- The middle base of the trapezium is the line segment connecting the midpoints of its legs.

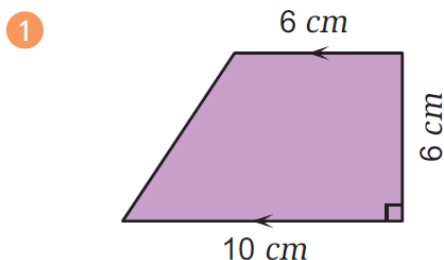
The length of the middle base
Thus: $= \frac{1}{2} \times \text{the sum of the lengths of the two parallel bases}$



Therefore: The area of the trapezium = length of the middle base $\times$ height

Example

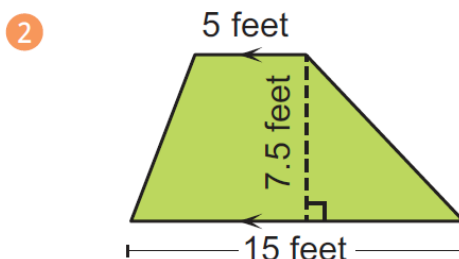
Calculate the area of the trapezium in each of the following shapes:



Solution

$$\begin{aligned} \textcircled{1} \therefore A &= \frac{1}{2} (b_1 + b_2) \times h \\ \therefore A &= \frac{1}{2} (6 + 10) \times 6 = 48 \end{aligned}$$

Thus, the area of the trapezium = 48 cm^2



Solution

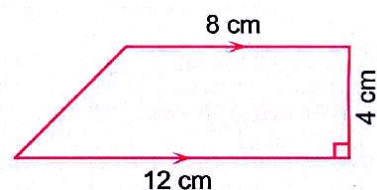
$$\begin{aligned} \textcircled{2} \therefore A &= \frac{1}{2} (b_1 + b_2) \times h \\ \therefore A &= \frac{1}{2} (5 + 15) \times 7.5 = 75 \end{aligned}$$

Thus, the area of the trapezium = 75 ft^2

Practicese

1

Find the area of the opposite trapezium.



2

A trapezium with a middle base length of 10 feet and a height of 5 feet. Find its area.

3

A trapezium with an area of 60 square centimeters and a height of 6 cm. Find the length of its middle base.

4

A trapezium with an area of 100 square yards, the length of its parallel bases are 5 yards and 15 yards. Find its height.

Choose the correct answer from the given ones :

1 If the area of a rhombus is 100 square units, what is the product of the lengths of its diagonals?

- (a) 25 (b) 50
(c) 100 (d) 200

2 A rhombus has an area of 20 square feet and the length of one of its diagonals is 5 feet. What is the length of the other diagonal?

- (a) 8 feet (b) 4 feet
(c) 10 feet (d) 15 feet

3 The area of a square with a side length of 6 cm the area of a square with a diagonal length of 8 cm.

- (a) < (b) > (c) =

4 If the area of a square is 450 square units, what is the length of its diagonal in length units?

- (a) 15 (b) 30
(c) 45 (d) 90

5 A trapezium has the sum of the lengths of its two parallel bases equal to 16 cm, and its height is 5 cm. What is its area in square centimeters?

- (a) 20 (b) 40 (c) 80 (d) 160

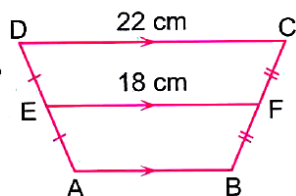
6 A trapezium the lengths of its two parallel bases are 15 cm and 11 cm. What is the length of its middle base?

- (a) 26 cm (b) 15 cm
(c) 13 cm (d) 11 cm

7 In the opposite figure:

What is the length of $\overline{AB}$ in centimeters?

- (a) 14 (b) 20
(c) 26 (d) 28



8 A trapezium has an area of 450 square inches and the lengths of its two parallel bases are 24 inches and 12 inches. What is its height?

- (a) 12.5 inches (b) 25 inches
(c) 36 inches (d) 52 inches

9 A trapezium has one of its parallel bases of length 15 cm and an area of 108 square centimeters. If its a height of length 8 cm, what is the length of the other base?

- (a) 15 cm (b) 4 cm
(c) 12 cm (d) 27 cm

10 A trapezium with a middle base length of x cm and a height is equal to half the length of its middle base. What is its area in square centimeters?

- (a) x^2 (b) $\frac{x^2}{2}$
(c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

Homework

1 - Find the area of each of the following shapes:

a A rhombus with a side length of 6 cm and a height of 5 cm.

b A rhombus with diagonal lengths of 16 inches and 30 inches.

c A square with a diagonal length of 10 meters.

d A square with a side length of 6 feet.

E A trapezium with parallel bases of length 6 cm and 8 cm and a height of 12 cm.

g A trapezium with a middle base length of 12 yards and a height of 8 yards.

Which has a greater area?

2 A square with a diagonal length of 12 cm or a rectangle with a length of 11 cm and a width of 7 cm.

3 For a rhombus with a side length of 10 feet, a height of 9.6 feet, and one diagonal length of 12 feet, **find the length of the other diagonal.**

4 A trapezium has an area of 45 square inches and a height of 5 inches. **Find the length of its middle base.**

5 A trapezium has an area of 175 square meters and the lengths of its two parallel bases are 14 meters and 21 meters. **Find its height.**

6 A trapezium has an area of 54 square centimeters and a height of 9 cm. If the length of its shorter base is equal to 4 cm, **find the length of its longer base.**

7 Find the length of the diagonal of the square whose area is equal to the area of a rhombus with diagonal lengths of 4 meters and 25 meters.

8 Two pieces of land are equal in area, the first in the shape of a square and the second in the shape of a rhombus with diagonal lengths 8 meters and 16 meters. **Find the perimeter of the square shaped land.**

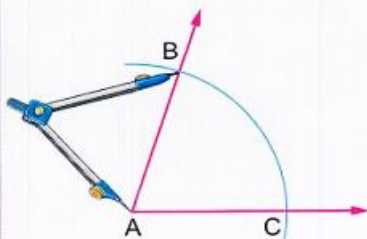
9 Two pieces of land are equal in area, the first in the shape of a rhombus with diagonal lengths of 8 meters and 27 meters, and the other in the shape of a trapezium its height is 6 meters. **Find the length of its middle base.**

Lesson 2: Geometrical Constructions

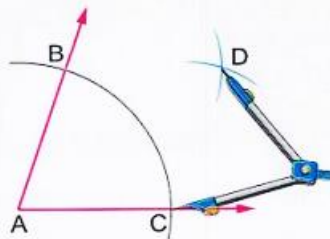
Bisecting an Angle

To bisect an angle as $\angle BAC$ by using a compass and ruler, you may follow these steps:

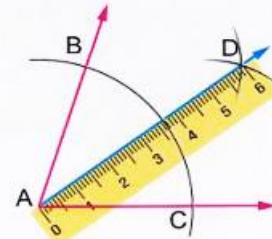
- 1 Place the needle of the compass at the vertex of the angle A, and draw an arc that intersects the sides of the angle at points B and C.



- 2 With the same compass span or any suitable span, place the compass needle at point C and draw an arc, then with the same span place it at point B and draw another arc that intersects the first arc at point D.



- 3 Draw $\overrightarrow{AD}$, which will be the bisector of angle BAC, and hence :
 $m(\angle BAD)$
 $= m(\angle DAC)$

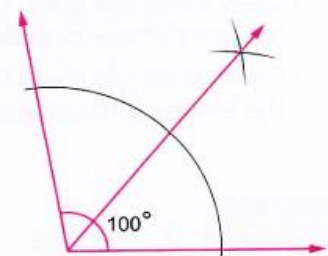


Example 1

Draw an angle of measure 100° , then bisect it using the ruler and compass. Verify by measuring that the bisection is accurate.

Solution

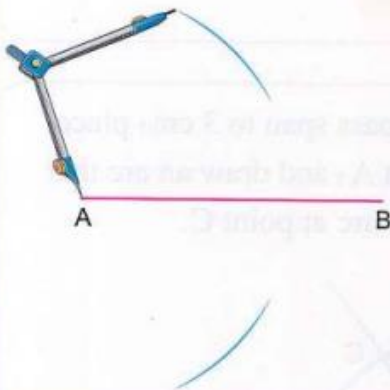
Using a protractor, you will find that both angles are equal in measure, each of measure 50° .



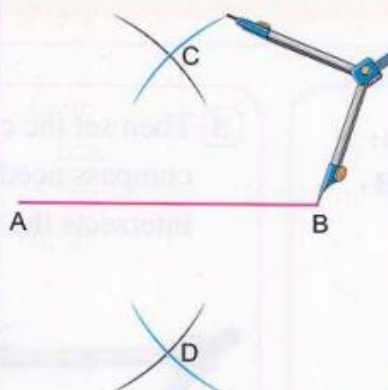
Bisecting a Line Segment

To bisect a line segment such as $\overline{AB}$ by using a compass and ruler, you may follow these steps:

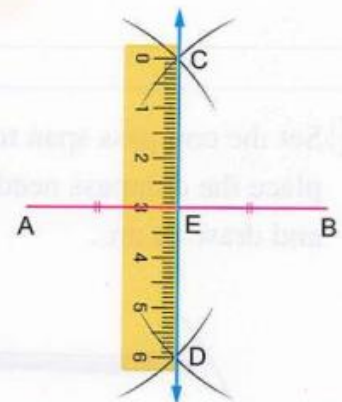
- 1 Set the compass span greater than half the length of $\overline{AB}$, then place the compass needle at A and draw two arcs in two different directions from A.



- 2 With the same compass span, place the compass needle at B and draw two arcs that intersect the previous arcs at points C and D.



- 3 Draw the straight line $\overleftrightarrow{CD}$ to intersect $\overline{AB}$ at the point E, thus the point E is the midpoint of $\overline{AB}$, i.e.: $AE = EB$



Example 2

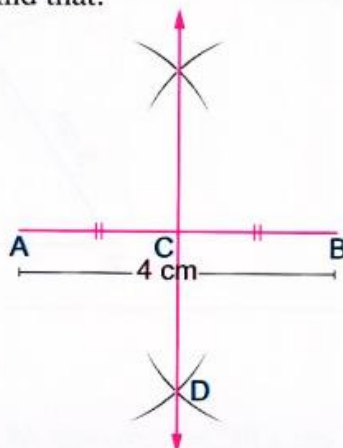
Draw $\overline{AB}$ of length 4 cm, and bisect it using a ruler and compass.

Verify the accuracy of the bisection by measurement.

Solution

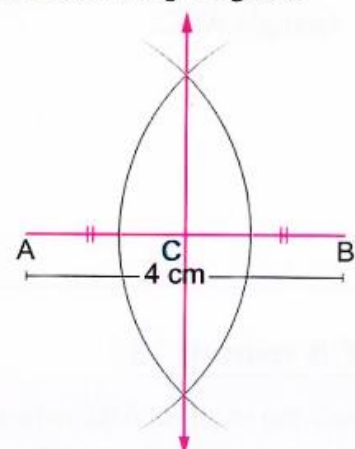
Using a ruler, you will find that:

$$BC = AC = 2 \text{ cm}$$



Note

You may draw the arcs as shown in the following diagram:



TRY it Yourself 2



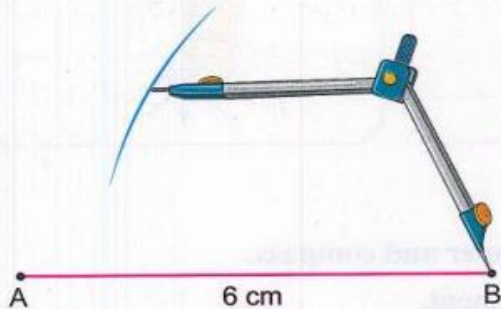
Drawing a Triangle**First** Drawing a triangle given the lengths of its sides

To draw the triangle ABC in which: the length of $\overline{AB} = 6$ cm, the length of $\overline{BC} = 5$ cm, and the length of $\overline{AC} = 3$ cm, follow these steps:

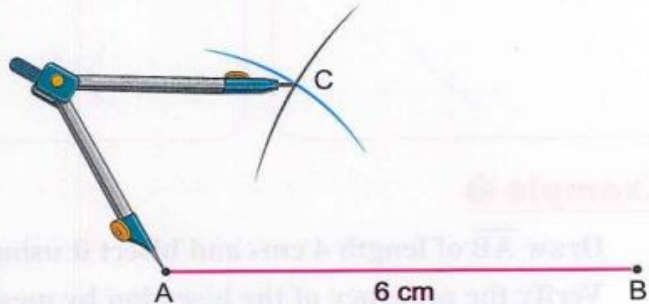
- 1 Use a ruler to draw a line segment $\overline{AB}$ of length 6 cm.



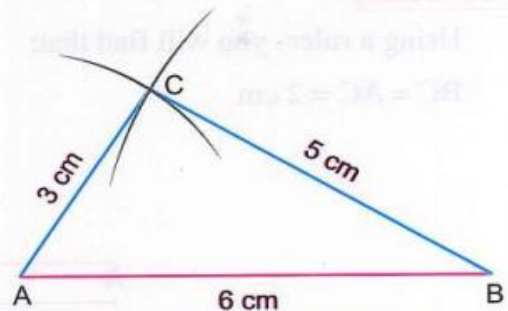
- 2 Set the compass span to 5 cm, place the compass needle at B, and draw an arc.



- 3 Then set the compass span to 3 cm, place compass needle at A, and draw an arc that intersects the first arc at point C.



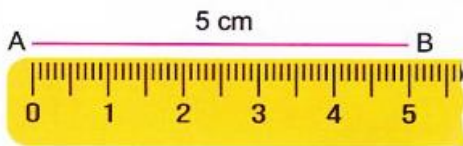
- 4 Draw $\overline{AC}$ and $\overline{BC}$ to obtain the required triangle ABC.



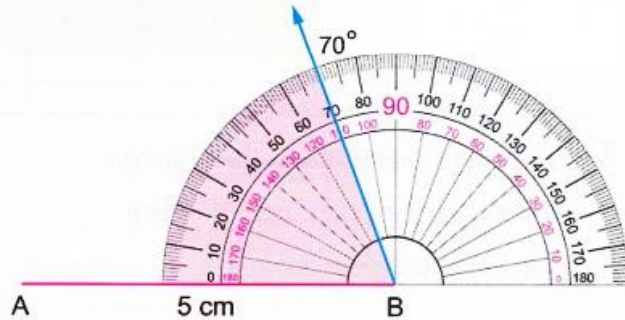
Second**Drawing a triangle given the lengths of two sides and the measure of the included angle between them**

To draw the triangle ABC in which: the length of $\overline{AB} = 5$ cm, the length of $\overline{BC} = 4$ cm, and $m(\angle ABC) = 70^\circ$, follow these steps:

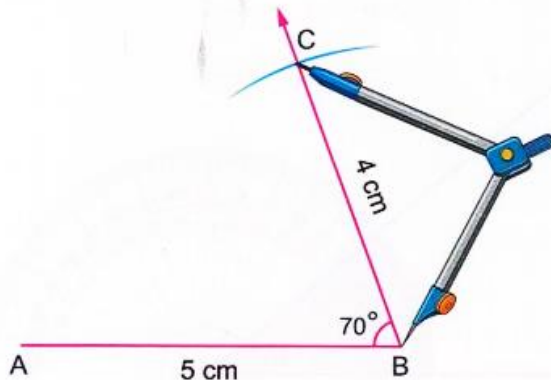
- 1 Use a ruler, to draw the line segment $\overline{AB}$ of length 5 cm.



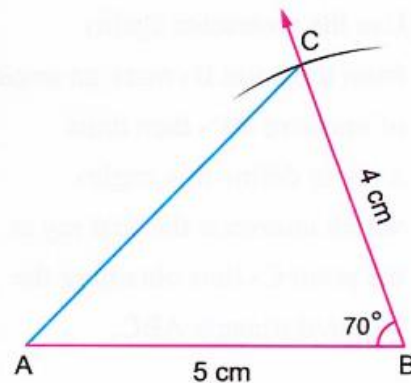
- 2 Using a protractor, from the point B, mark an angle of measure 70° , then draw a ray that defines the angle.



- 3 Set the compass span to 4 cm, then place its needle at B and draw an arc to intersect the drawn ray at the point C, thus the length of $\overline{BC} = 4$ cm.



- 4 Draw $\overline{AC}$ to obtain the required triangle ABC.

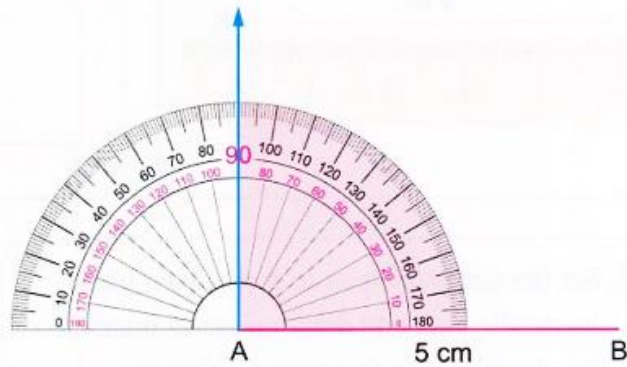
**Third****Drawing a triangle given the measures of two angles and the length of the side drawn between their vertices.**

To draw the triangle ABC, in which: the length of $\overline{AB} = 5$ cm, $m(\angle A) = 90^\circ$, and $m(\angle B) = 40^\circ$, follow these steps:

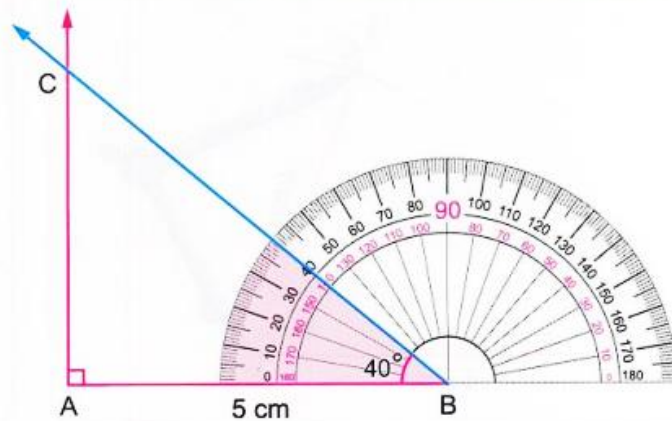
- 1 Use a ruler to draw the line segment $\overline{AB}$ of length 5 cm.



- 2 Using a protractor, from the point A, mark an angle of measure 90° , then draw a ray to define the angle.



- 3 Use the protractor again, from the point B, mark an angle of measure 40° , then draw a ray to define this angle, which intersects the first ray at the point C, thus obtaining the required triangle ABC.

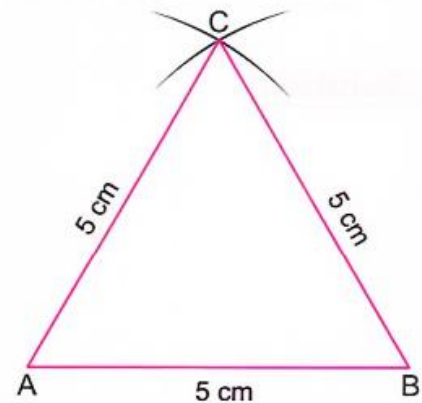


Example ③

Draw the equilateral triangle ABC with a side length of 5 cm.

Solution

Draw the triangle ABC using the lengths of its three sides:



Exercises

- 1  Draw an angle of measure 45° , then bisect it using a ruler and compass. Verify by measuring that the bisection is accurate.
- 2  Draw $\angle ABC$ of measure 120° , then bisect it using a ruler and compass by the bisector $\overrightarrow{BD}$, showing the steps of the solution. Verify by using a protractor that:
 $m(\angle ABD) = m(\angle CBD)$
- 3 Draw $\angle ABC$ of measure 85° , then bisect it using a ruler and compass with the bisector $\overrightarrow{BD}$, showing the steps of the solution. Verify by using a protractor that: $m(\angle ABD) = m(\angle CBD)$
- 4 Draw $\angle XYZ$ of measure 145° , then bisect it using a ruler and compass with the bisector $\overrightarrow{YL}$, showing the steps of the solution. Verify by using a protractor that:
 $m(\angle XYL) = m(\angle LYZ)$



- 8  Draw $\overline{AB}$ of length 5 cm and bisect it using a ruler and compass. Verify by measuring that the bisection is accurate.
- 9  Draw a line segment $\overline{AB}$ of length 7 cm, then bisect it using a ruler and compass at point C, showing the steps of the solution. Verify by using a ruler that C is the midpoint of $\overline{AB}$.



- 12  Draw $\triangle XYZ$ where $XY = 6$ cm, $YZ = 4$ cm, $XZ = 5$ cm, then determine by measuring the type of the triangle according to the measures of its angles.
- 13 Draw the triangle LMN where $MN = LM = 6$ cm, $LN = 5$ cm and determine, by measuring, the type of triangle according to the measures of its angles.
- 14  Draw the triangle ABC where: $AB = 7$ cm, $BC = 9$ cm, $AC = 4$ cm and determine, by measuring, its type according to the measures of its angles.
- 15  Draw the equilateral triangle ABC, the length of each side is 6 cm.

- 22  Draw the triangle XYZ where $XY = XZ = 6$ cm , $ZY = 8$ cm. Then bisect both $\angle Y$ and $\angle Z$ with bisectors intersecting at point M. Verify by measuring that: $MY = MZ$
- 23 Draw the triangle LMN where $MN = LM = 10$ cm, $LN = 12$ cm. Then bisect $\angle M$ with the bisector $\overrightarrow{MD}$ which intersects $\overline{LN}$ at D. Determine by measuring $m(\angle MDN)$
- 24 Draw the triangle XYZ where $XY = 7$ cm, $m(\angle X) = 48^\circ$, $m(\angle Y) = 52^\circ$. Then bisect both $\overline{YZ}$ and $\overline{XZ}$ at points D and E respectively. Determine the length of $\overline{DE}$.
- 25 Draw the triangle RST where $RS = 4.5$ cm, $ST = 6$ cm, $m(\angle S) = 90^\circ$, then bisect $\overline{RT}$ at the point Z. Then, determine the ratio $SZ : RT$.
- 26  Using geometrical tools, draw the triangle ABC where the length of $\overline{AB}$ equals 4 cm , the length of $\overline{BC}$ equals 3 cm , and $m(\angle B) = 90^\circ$. Then bisect $\overline{AC}$ at point D.
Is $BD = \frac{1}{2} AC$?



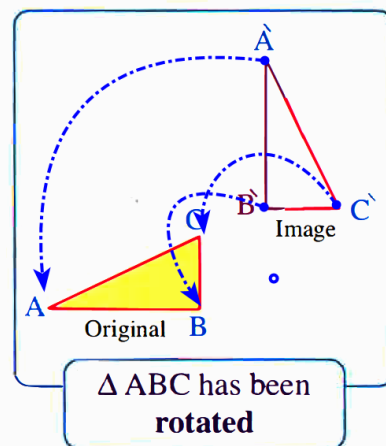
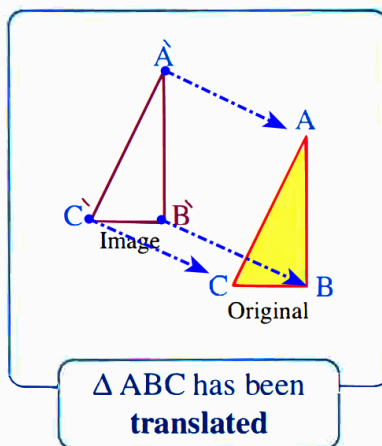
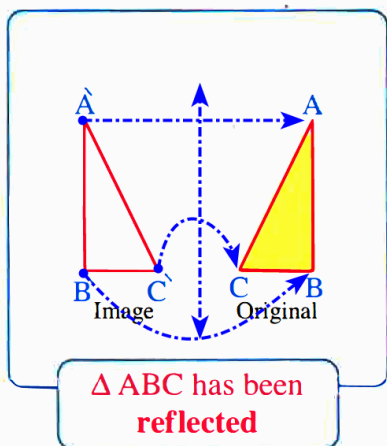
lesson 3-3: Geometrical Transformations

In this lesson, you will recognize the meaning of the geometric transformation, also recognize quickly three types of them:

- ① Reflection.
- ② Translation.
- ③ Rotation.

The concept of the geometric transformations

In each of the following figures , notice the image of ΔABC :



From the previous' we deduce that:

- ① The point A has been transferred to A' , The point B has been transferred to B' , The point C has been transferred to C'
- ② all the points of a geometrical figure have moved according to a certain system

Example (1):

Draw the image of ΔABC where $A(1, 1)$, $B(4, 1)$, $C(1, 5)$ according to each of the following transformations, $(X, y) \Rightarrow (-X, y)$

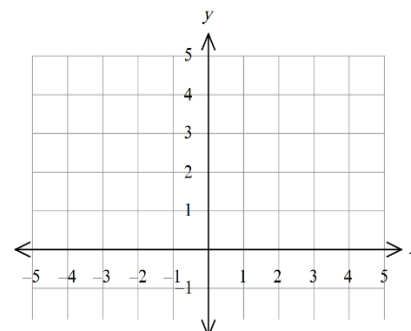
Solution

$$A(1, 1) \xrightarrow{(-X, y)} \hat{A}(-1, 1)$$

$$B(4, 1) \xrightarrow{(-X, y)} \hat{B}(-4, 1)$$

$$C(1, 5) \xrightarrow{(-X, y)} \hat{C}(-1, 5)$$

The transformation is **reflection**.



Example (2):

Draw the image of ΔABC where $A(1, 1)$, $B(4, 1)$, $C(1, 5)$ according to each of the following transformations, $(X, y) \Rightarrow (-y, X)$

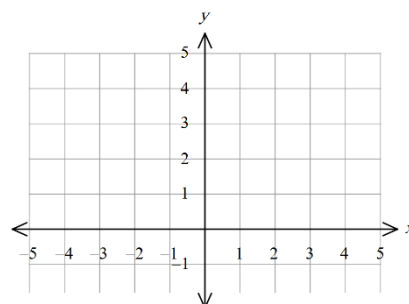
Solution

$$A(1, 1) \xrightarrow{(-y, x)} \hat{A}(\dots, \dots)$$

$$B(4, 4) \xrightarrow{(-y, x)} \hat{B}(\dots, \dots)$$

$$C(1, 5) \xrightarrow{(-y, x)} \hat{C}(\dots, \dots)$$

The transformation is

**Example (3):**

Draw the image of A ABC where

A(1, 1), B(4, 1), C(1, 5) according to each of the following transformations,

$$(X, y) \Rightarrow (X, y-5)$$

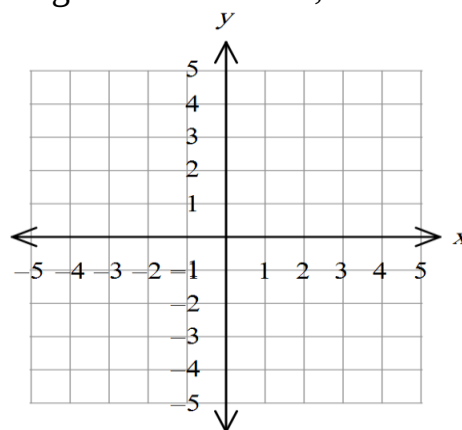
Solution

$$A(1, 1) \xrightarrow{(X, y-5)} \hat{A}(\dots, \dots)$$

$$B(4, 1) \xrightarrow{(X, y-5)} \hat{B}(\dots, \dots)$$

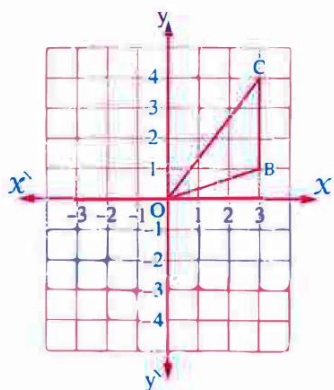
$$C(1, 5) \xrightarrow{(X, y-5)} \hat{C}(\dots, \dots)$$

The transformation is

**Exercise**

Map each of the following shapes due to the geometric transformation above it, then describe its type :

$$\textcircled{1} (x, y) \longrightarrow (-x, -y)$$



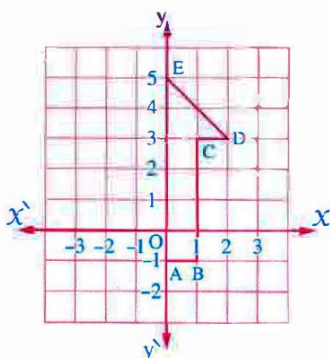
$$O(0,0) \longrightarrow O(\dots, \dots)$$

$$B(3,1) \longrightarrow B'(\dots, \dots)$$

$$C(3,4) \longrightarrow C'(\dots, \dots)$$

.....

$$\textcircled{2} (x, y) \longrightarrow (-x, y)$$



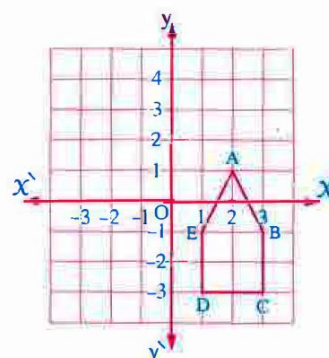
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$$\textcircled{3} (x, y) \longrightarrow (x, y+2)$$



.....

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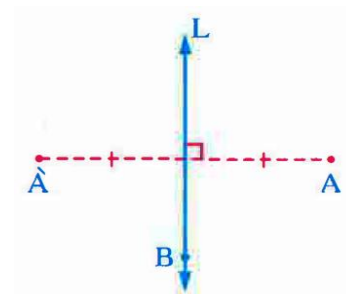
.....

(1) Reflection

Definition of reflection in a straight line:

Reflection in the straight line L maps each point A to the point in the same plane such that :

- 1) If $A \notin L$, then the straight line L is the perpendicular bisector to the line segment $\overline{AA'}$
- 2) If $B \in L$, then B is reflected onto itself

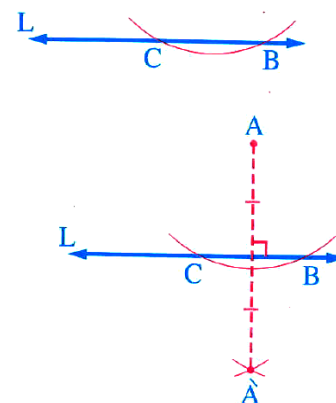


Finding the image of a point by reflection in a given straight line

- To find $\hat{A}$ which is the image of A by reflection in the straight line L , we do as follows :

- 1 Draw an arc of a circle with centre A to cut L at B and C
- 2 With the same radius length taking B and C as centres , draw two arcs in the other side of the straight line L to intersect at $\hat{A}$, then $\hat{A}$ is the image of A by reflection in L

A



Check by measuring that :

$\overline{AA} \perp L$ and L bisects $\overline{AA}$

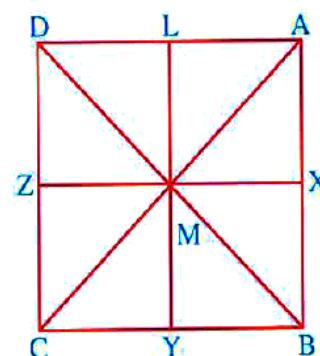
Example:

In the opposite figure :

$ABCD$ is a square. M is the point of intersection of its diagonals. X , Y , Z and L are the midpoints of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Complete the following :

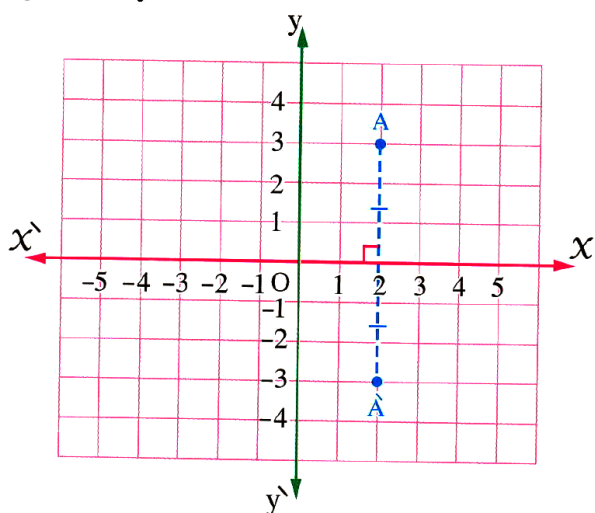
- 1 The image of the point A by reflection in $\overleftrightarrow{LY}$ is
- 2 The image of the $\overline{AM}$ by reflection in $\overleftrightarrow{XM}$ is
- 3 The image of the $\triangle ALM$ by reflection in $\overleftrightarrow{LY}$ is
- 4 The image of the $\triangle ALM$ by reflection in $\overleftrightarrow{AM}$ is
- 5 The image of the $\triangle AMB$ by reflection in $\overleftrightarrow{LY}$ is



Reflection in the Cartesian coordinates plane:

Reflection in the x -axis

- To find the image of the point A (2 , 3) by reflection in $\overleftrightarrow{XX}$ (the X -axis) : draw $\overline{AA'}$ such that $\overleftrightarrow{XX}$ is the line of symmetry of it.



Then we find that :

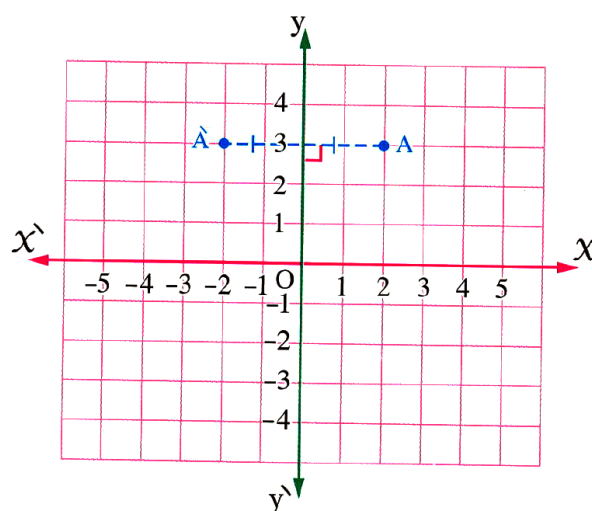
$$A(2, 3) \longrightarrow \hat{A}(2, -3)$$

- i.e. The reflection in the X -axis changes the sign of the 2nd projection (y-coordinates)

$$A(X, y) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} \hat{A}(X, -y)$$

Reflection in the y -axis

- To find the image of the point A (2 , 3) by reflection in $\overleftrightarrow{yy}$ (the y -axis) : draw $\overline{AA'}$ such that $\overleftrightarrow{yy}$ is the line of symmetry of it.



Then we find that :

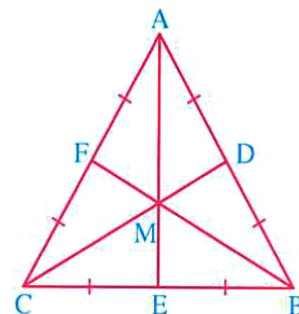
$$A(2, 3) \longrightarrow \hat{A}(-2, 3)$$

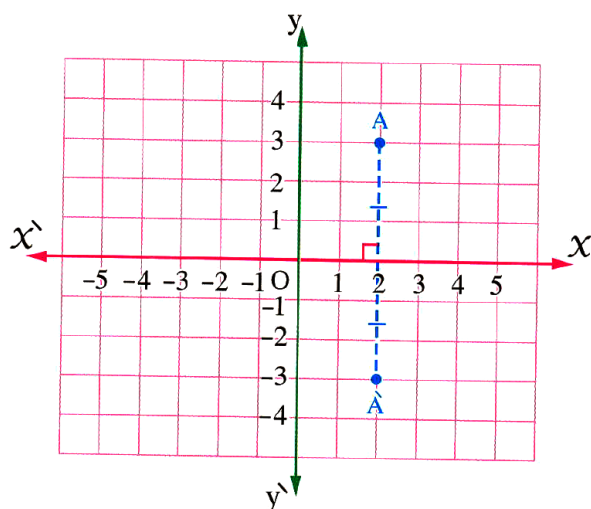
- i.e. The reflection in the y -axis changes the sign of the 1st projection (X-coordinates)

$$A(X, y) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} \hat{A}(-X, y)$$

Complete :

- The axes of symmetry of $\triangle ABC$ are
- $\overline{AB}$ is the reflected image of $\overline{AC}$ by reflection in
- The reflected image of $\overline{AF}$ by reflection in $\overleftrightarrow{BF}$ is , and the reflected image of $\overline{CF}$ in $\overleftrightarrow{AE}$ is
- The reflected image of $\triangle AMD$ by reflection in $\overleftrightarrow{AE}$ is
 $\therefore m(\angle AMD) = m(\angle \dots\dots\dots)$, because reflection in a line reserves
- The reflected image of $\triangle AMB$ by reflection in $\overleftrightarrow{AE}$ is





Then we find that :

$$A(2, 3) \longrightarrow \hat{A}(2, -3)$$

i.e. The reflection in the x -axis changes the sign of the 2nd projection (y-coordinates)

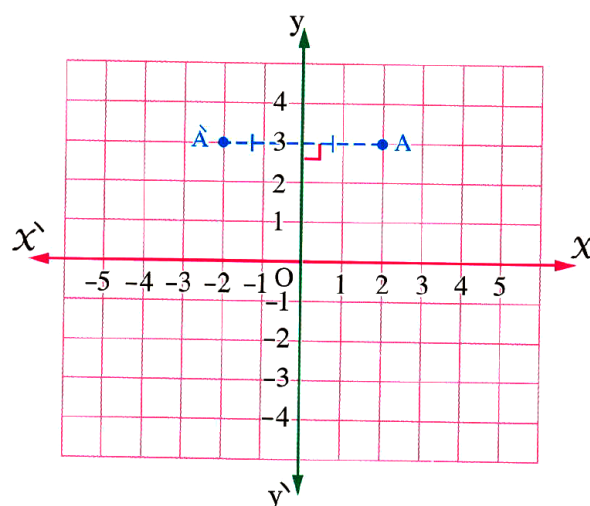
$$A(X, y) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} \hat{A}(X, -y)$$

For example:

- $(2, 4) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (2, -4)$
- $(-2, -1) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (-2, 1)$
- $(-5, 3) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (-5, -3)$
- $(7, -2) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (7, 2)$

Remarks

- 1 The image of the point $(X, 0)$ by reflection in the x -axis is itself because it lies on the x -axis
For example: $(5, 0) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (5, 0)$
- 2 The image of the point $(0, y)$ by reflection in the y -axis is itself because it lies on the y -axis
For example: $(0, -3) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (0, -3)$
- 3 The image of the point $(0, 0)$ by reflection in the x -axis or the y -axis is itself because it lies on both two axes.



Then we find that :

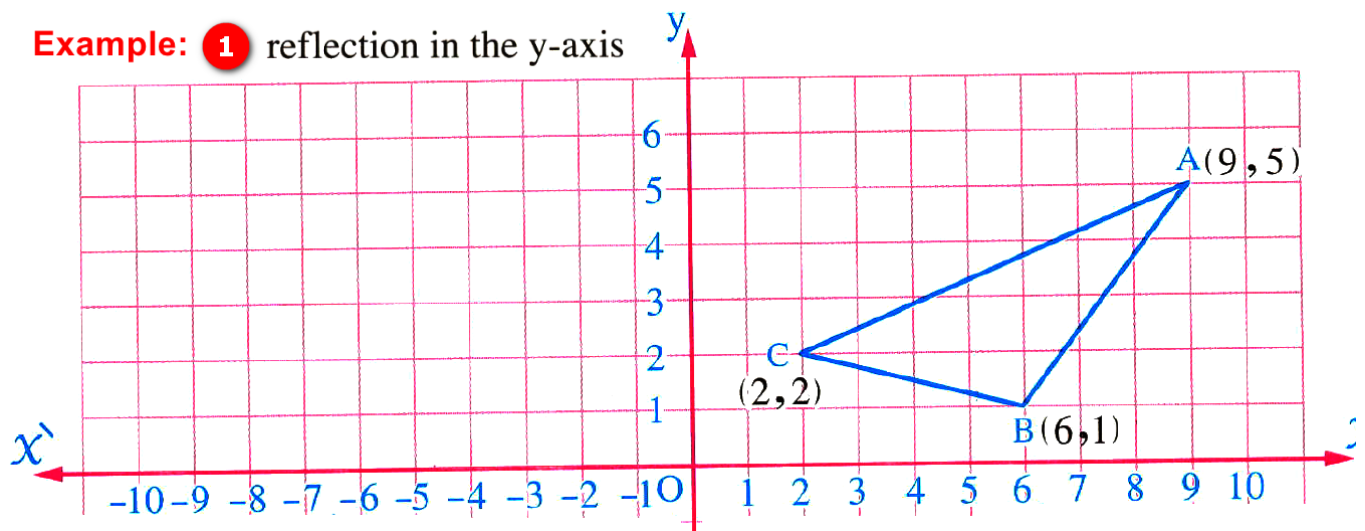
$$A(2, 3) \longrightarrow \hat{A}(-2, 3)$$

i.e. The reflection in the y -axis changes the sign of the 1st projection (x -coordinates)

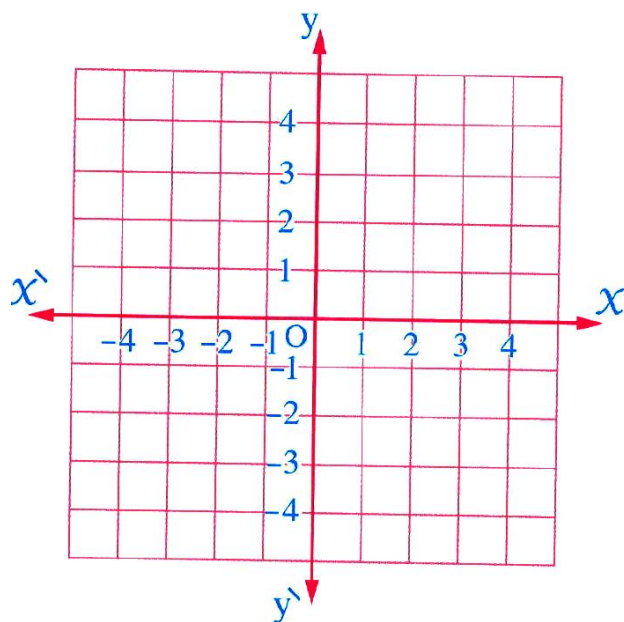
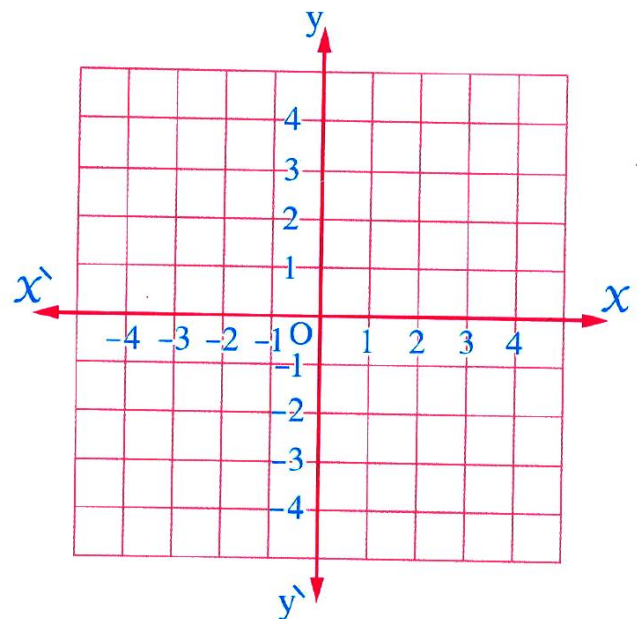
$$A(X, y) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} \hat{A}(-X, y)$$

For example:

- $(2, 4) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (-2, 4)$
- $(-2, -1) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (2, -1)$
- $(-5, 3) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (5, 3)$
- $(7, -2) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (-7, -2)$

Example: 1 reflection in the y-axis**Example : 2**

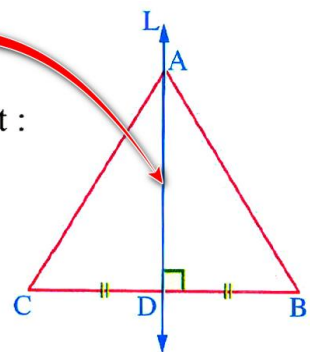
Draw $\triangle ABC$ where A (1 , 1) , B (4 , 1) and C (3 , 3) , then draw its image by reflection in :

1 The x-axis**2** The y-axis**Line of symmetry :**

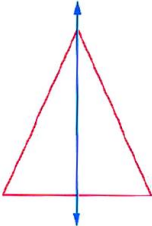
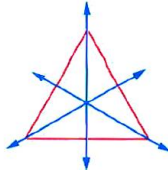
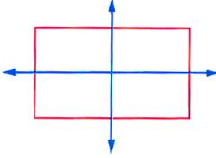
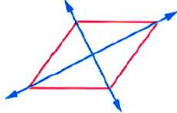
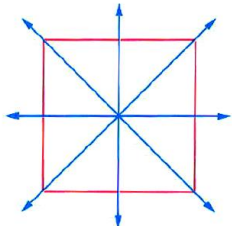
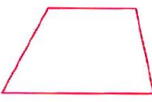
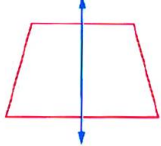
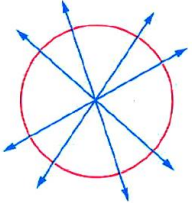
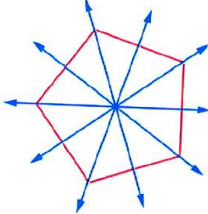
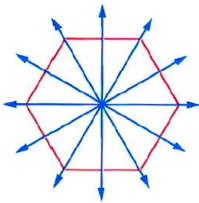
ABC is a triangle , $\overrightarrow{AD} \perp \overline{BC}$, D is the midpoint of $\overline{BC}$, we find that :

- The image of A by reflection in L is itself (A)
- The image of B by reflection in L is C
- The image of C by reflection in L is B

**Line of symmetry
of $\triangle ABC$**



The axes of symmetry of some geometric figures

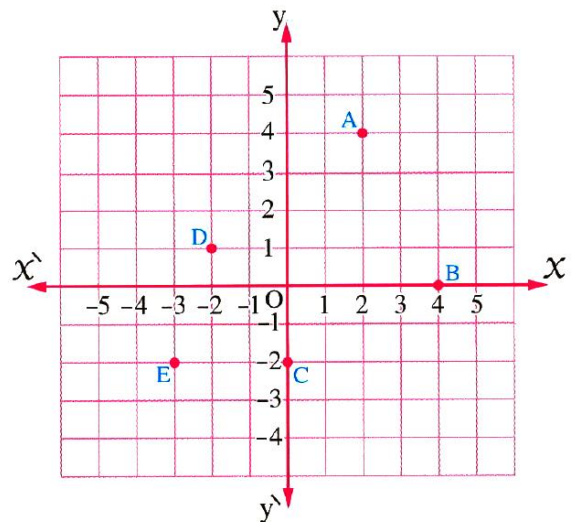
The figure			
Number of axes of symmetry	1	3	Zero (does not exist)
The figure			
Number of axes of symmetry	Zero (does not exist)	2	2
The figure			
Number of axes of symmetry	4	Zero (does not exist)	1
The figure			
Number of axes of symmetry	An infinite number	5	6

Example : 3

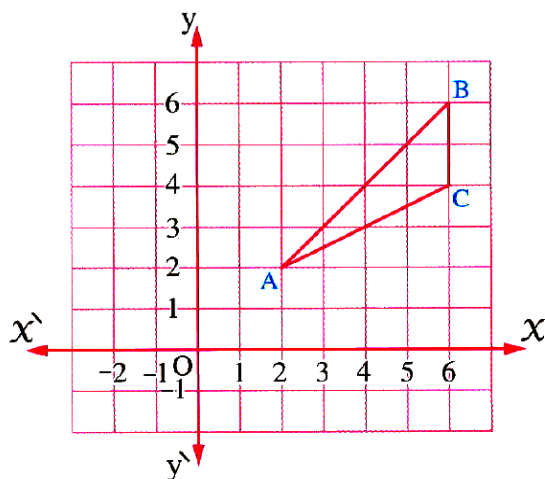
In the opposite figure :

Write the coordinates of the image of each point by reflection in :

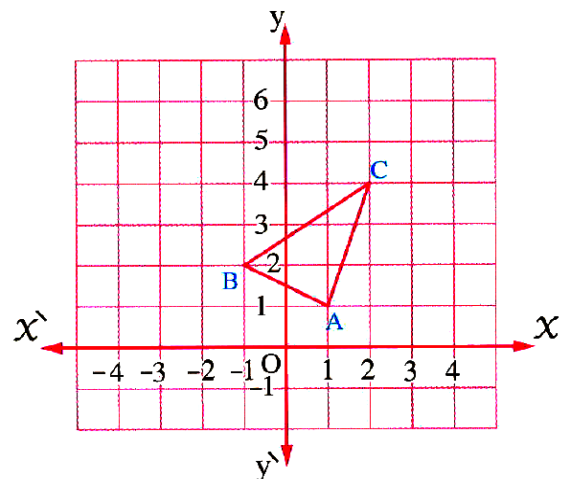
- 1 The x -axis
- 2 The y -axis

**Example: 4**

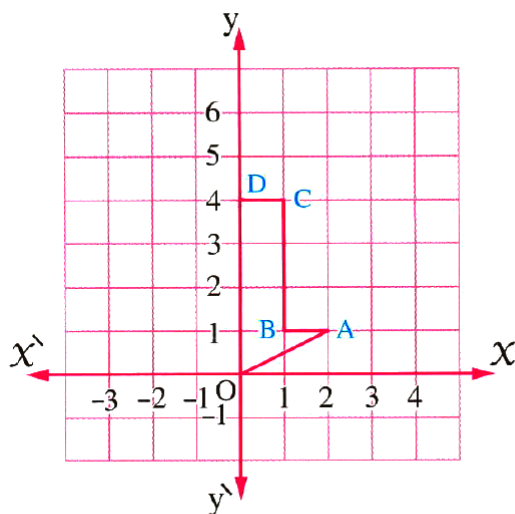
Copy each of the following figures on a lattice and draw the image of the figure by a geometric transformation as shown below each figure , then write the coordinates of each vertex of the figure.



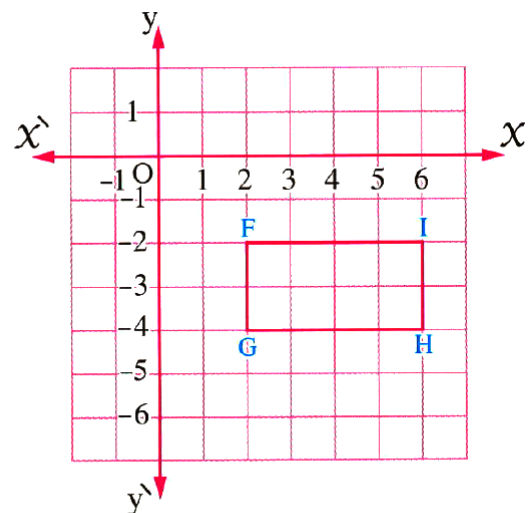
reflection in the x -axis



reflection in the y -axis



reflection in the y -axis



reflection in $\overleftrightarrow{FG}$

Exercises

(1) Complete each of the following :

- 1 The image of the point $(3, -2)$ by reflection in y-axis is
- 2 The image of the point $(3, -4)$ by reflection in the origin point is
- 3 The square is a rhombus with two diagonals
- 4 The polygon that contains a reflex angle is called

(2) Complete the following :

- 1 The reflection in a plane reserves :,,,
- 2 If the reflection in a straight line transforms the figure to itself , then this straight line is called

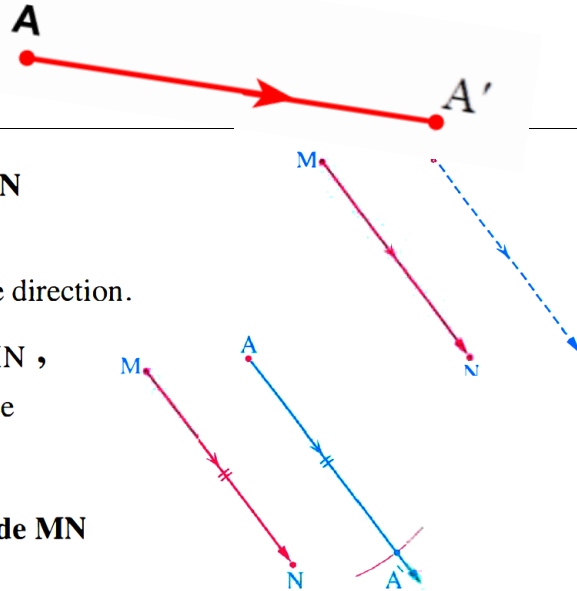
(3) Choose the correct answer from those given :

- a The image of the triangle by reflection in the origin point is
(a) triangle. (b) square. (c) point. (d) straight line.
- b If ABC is a triangle in which $m(\angle A) = 4x^\circ$, $m(\angle B) = 2x^\circ$, $m(\angle C) = 3x^\circ$, then $\angle A$ is angle.
(a) an acute (b) a right (c) an obtuse (d) a reflex
- c The point $(2, -3)$ is the image of the point by reflection in the origin point.
(a) $(-2, -3)$ (b) $(-2, 3)$ (c) $(2, 3)$ (d) $(3, 2)$
- d
- e If the image of the point (a, b) by reflection in the origin point is the point (x, y) and $a > b$, then x y
(a) $>$ (b) $=$ (c) $\geq$ (d) $<$
- f The image of the point $(-3, 0)$ is itself by reflection in
(a) x-axis. (b) y-axis. (c) origin point. (d) axis of symmetry.
- g If the image of the point $(a - 3, 7)$ by reflection in y-axis is itself , then $a =$
(a) 10 (b) 3 (c) -3 (d) 7

(2) Translation

Translation: is a geometrical transformation which maps each point A in the plane to another point A' in the same plane with a constant distance in a certain direction.

Translation in the plane:



- To find $\hat{A}$ which is the image of A by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :

- 1 Draw from A a ray parallel to $\overrightarrow{MN}$ and in the same direction.
- 2 By the compasses in A as a centre with radius $= MN$, draw an arc to intersect the ray drawn from A at the point $\hat{A}$ ($A\hat{A} = MN$ and $\overline{A\hat{A}} \parallel \overline{MN}$)

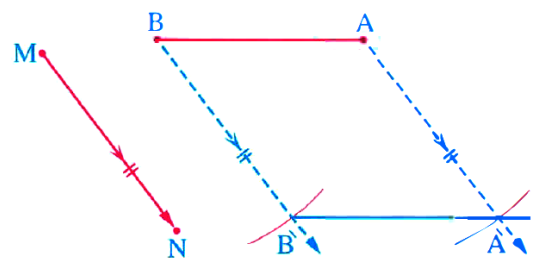
- Then $\hat{A}$ is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$

- Then $\hat{A}$ is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$

Finding the image of a line segment by given Translation:

- To find the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :

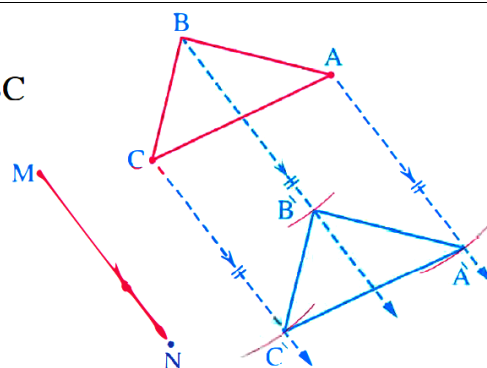
- 1 Find the image of the point A by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before, say $\hat{A}$
- 2 Similarly, we find the image of the point B by translation MN in the direction of $\overrightarrow{MN}$, say $\hat{B}$
- 3 Draw $\overline{\hat{A}\hat{B}}$ to be the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$



Check that : $AB = \hat{A}\hat{B}$ and $\overline{AB} \parallel \overline{\hat{A}\hat{B}}$

Finding the image of a Triangle by given Translation:

- 1 Find the image of each vertex of the vertices of $\triangle ABC$ by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before (say $\hat{A}$ for A , $\hat{B}$ for B and $\hat{C}$ for C)
- 2 Draw $\overline{\hat{A}\hat{B}}$, $\overline{\hat{B}\hat{C}}$ and $\overline{\hat{C}\hat{A}}$ then $\triangle \hat{A}\hat{B}\hat{C}$ is the image of $\triangle ABC$ by translation MN in the direction of $\overrightarrow{MN}$



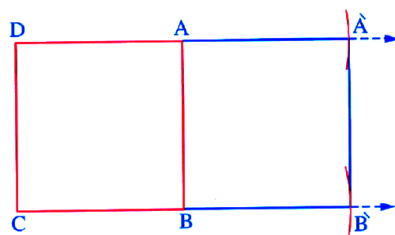
Check that :

- $AB = \hat{A}\hat{B}$, $BC = \hat{B}\hat{C}$ and $CA = \hat{C}\hat{A}$
- $m(\angle A) = m(\angle \hat{A})$, $m(\angle B) = m(\angle \hat{B})$, $m(\angle C) = m(\angle \hat{C})$

Properties of translation

Draw the square ABCD whose side length is 3 cm., then draw its image by translation AB in the direction $\overrightarrow{DA}$

The square $\hat{A}\hat{B}\hat{B}\hat{A}$ is the image of the square ABCD by translation AB in the direction $\overrightarrow{DA}$



Notice that :

- 1 $\hat{A}\hat{B} = AB$, $AB = DC$

i.e.

Translation reserves the lengths of the line segments.

- 2 $m(\angle \hat{A}) = m(\angle BAD)$, $m(\angle \hat{B}) = m(\angle CBA)$

i.e.

Translation reserves the measures of the angles.

- 3 From the square ABCD : $\overline{AB} \parallel \overline{DC}$, from the square $\hat{A}\hat{B}\hat{B}\hat{A}$: $\overline{\hat{A}\hat{B}} \parallel \overline{\hat{A}\hat{B}}$
 $\therefore$ The images of the two parallel line segments are also two parallel line segment.

i.e.

Translation reserves the parallelism.

- 4 The reading of the square ABCD is in the clockwise direction and the reading of the square $\hat{A}\hat{B}\hat{B}\hat{A}$ is in the clockwise direction also.

i.e.

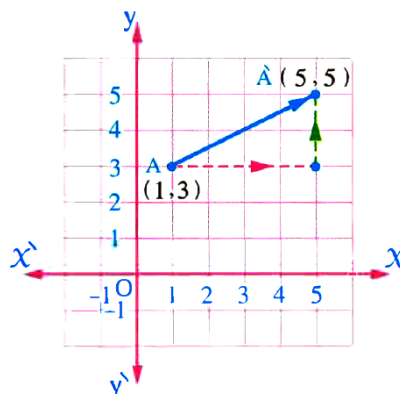
Translation reserves the orientation of the vertices of the figure.

Translation in the Cartesian plane:

If A (1 , 3) is a point in the orthogonal coordinates plane and to find its image $\hat{A}$ by translation with magnitude 4 length units in the direction of $\overrightarrow{OX}$ followed by a translation with magnitude 2 length units in the direction of $\overrightarrow{OY}$

From the graph , we get $\hat{A}$ to be the point (5 , 5)

i.e. $\hat{A}$ (1 + 4 , 3 + 2)



According to this :

Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement a in the direction of the X-axis followed by a displacement b in the direction of the y-axis

i.e. The image of the point A (X , y) $\longrightarrow$ the point $\hat{A}$ (X + a , y + b)

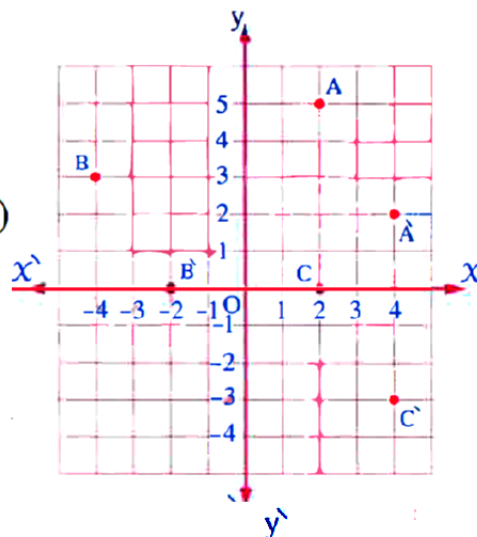
Example 1

Find the images of the points A (2 , 5) , B (− 4 , 3) and C (2 , 0) by translation (X , y) $\longrightarrow$ (X + 2 , y − 3)

Solution

$\therefore$ (X , y) $\longrightarrow$ (X + 2 , y − 3) , then :

- The image of A (2 , 5) is $\hat{A}$ (2 + 2 , 5 − 3)
i.e. $\hat{A}$ (4 , 2)
- The image of B (− 4 , 3) is $\hat{B}$ (− 4 + 2 , 3 − 3)
i.e. $\hat{B}$ (− 2 , 0)
- The image of C (2 , 0) is $\hat{C}$ (2 + 2 , 0 − 3)
i.e. $\hat{C}$ (4 , − 3)



Notice that :

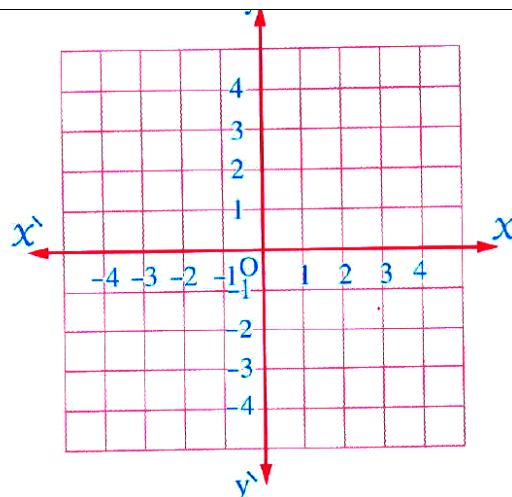
The translation (X , y) $\longrightarrow$ (X + 2 , y − 3) transforms each point to another point by a right horizontal displacement of 2 units and a vertical displacement of 3 units downwards.

Exercises

(1)

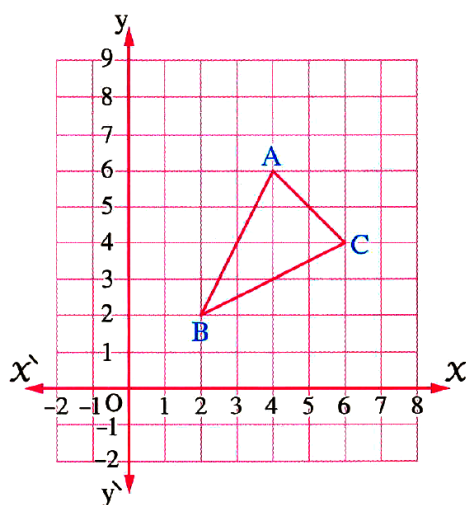
On a square lattice, draw $\triangle ABC$ where
 $A(-3, 2)$, $B(-1, 1)$, $C(-2, 0)$
 , then draw its image by translation :

$$(x, y) \longrightarrow (x + 2, y + 1)$$

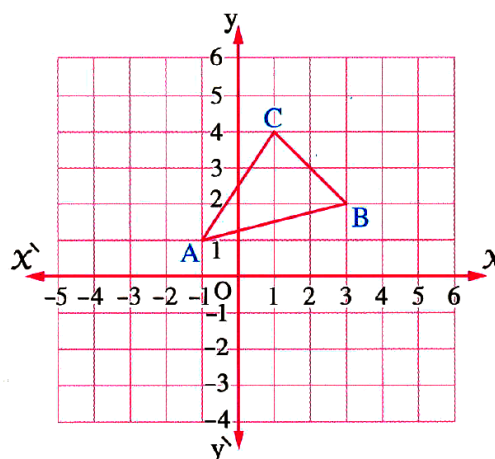


(2)

Draw the image of each of the following figures by the translation shown under each figure :



2



(3)

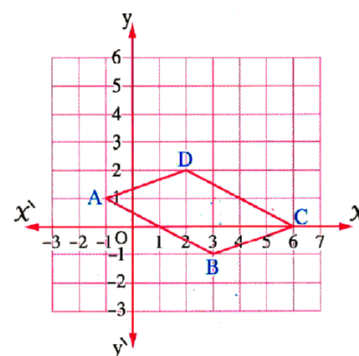
Copy the graph, then draw the image of the parallelogram ABCD under each of the following translations :

1 $(x, y) \longrightarrow (x + 5, y + 2)$

2 $(x, y) \longrightarrow (x - 8, y - 1)$

3 $(x, y) \longrightarrow (x + 2, y - 4)$

4 $(x, y) \longrightarrow (x - 4, y + 2)$



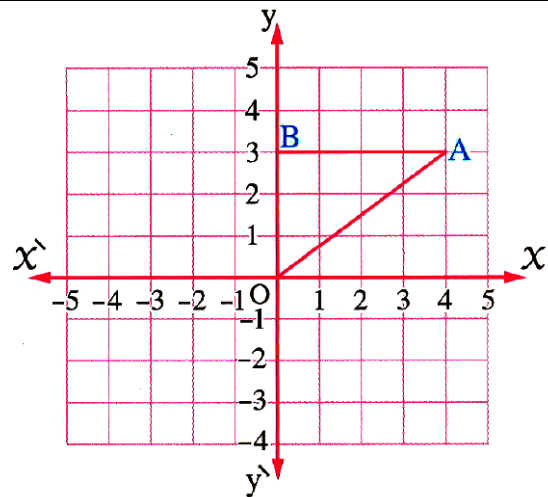
Notice that :

The translation with magnitude $\overrightarrow{MN}$ in the direction of $\overrightarrow{MN}$ where $M(4, 2)$ and $N(1, 4)$ is equivalent to :

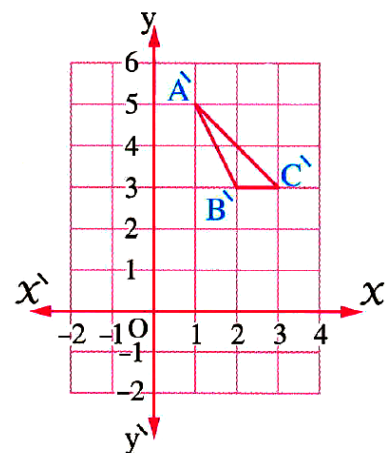
- A horizontal displacement (in the x -axis direction) from 4 to 1 = $1 - 4 = -3$
- A vertical displacement (in the y -axis direction) from 2 to 4 = $4 - 2 = 2$

i.e. The rule of translation is $(x, y) \longrightarrow (x - 3, y + 2)$

- (4) Draw the image of $\triangle AOB$ by the translation of magnitude = AO and in the direction of $\overrightarrow{AO}$

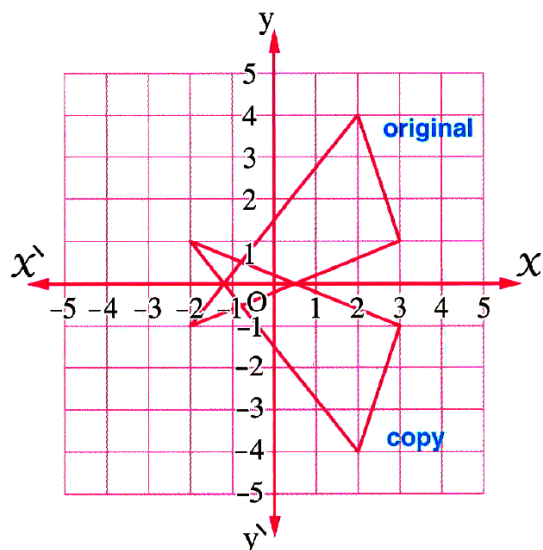
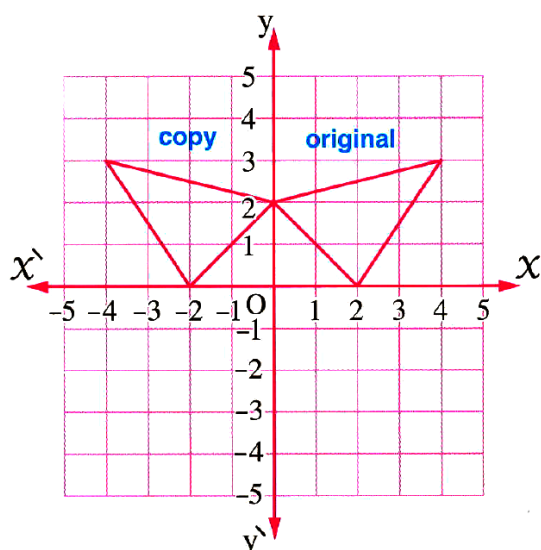


- (5) In the opposite figure :
Copy the graph , then draw the triangle ABC whose image is $\triangle A'B'C'$ by the translation
 $(x, y) \longrightarrow (x + 2, y + 3)$



- (6) State whether the graph shows a reflection or a translation :

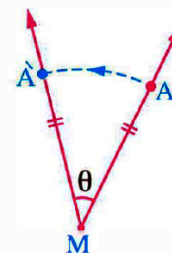
- 1 Name the line of reflection.
- 2 Describe the translation.



Definition of rotation:

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point $\hat{A}$ in the same plane such that $m(\angle AMA) = \theta$, $MA = M\hat{A}$ this rotation is denoted by $R(M, \theta)$ where :

- M is the centre of rotation.
- θ is the measure of the angle of rotation.

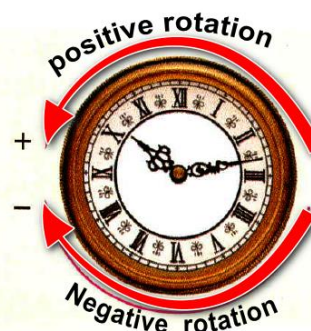


According to this definition, the rotation is determined completely if we know the following elements

- 1 The centre of the rotation.
- 2 The measure of the angle of the rotation (θ)
- 3 The direction of rotation.

Remark

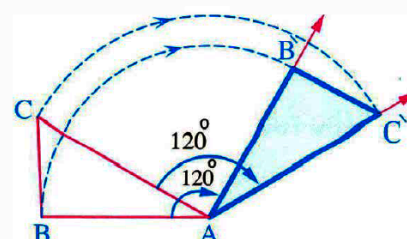
The measure of rotation angle is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.



Finding the image of a polygon:

The opposite figure shows how to find the image of $\triangle ABC$ by the rotation $R(A, -120^\circ)$ by finding the image of each vertex of its vertices, then $\triangle A\hat{B}\hat{C}$ is the image of $\triangle ABC$ by rotation $R(A, -120^\circ)$

Notice that : $\triangle A\hat{B}\hat{C} = \triangle ABC$

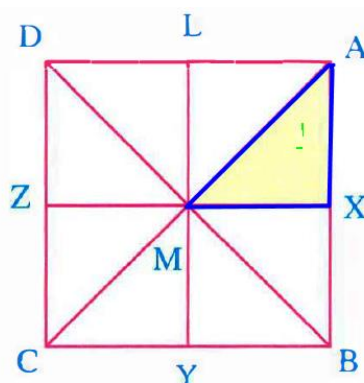


Remark

From the previous figure, the image of the point A by rotation $R(A, -120^\circ)$ is itself because it is the centre of rotation.



Properties of rotation:



We notice that :

- $DL = AX$, $LM = XM$ and $DM = AM$

i.e.

Rotation in the plane reserves the lengths of the line segments.

- $m(\angle DLM) = m(\angle AXM)$,
 $m(\angle LDM) = m(\angle XAM)$ and
 $m(\angle DML) = m(\angle AMX)$

i.e.

Rotation in the plane reserves the measures of the angles.

- Reading $\triangle AXM$ is in the clockwise direction and reading its image $\triangle DLM$ is in the clockwise direction also.

i.e.

Rotation in the plane reserves the orientation of the vertices of the figure.

- $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

i.e.

Rotation in the plane reserves the parallelism.

- $Y \in \overline{BC}$
and L (The image of Y) $\in \overline{AD}$

i.e.

Rotation in the plane reserves the betweenness.

- B, Y, C are collinear , D, L, A are also collinear.

i.e.

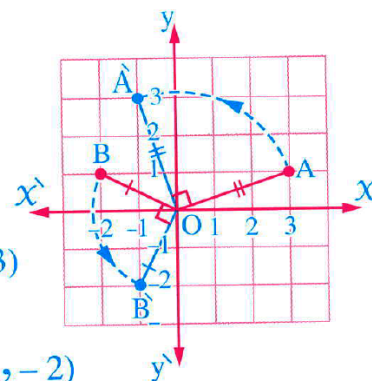
Rotation in the plane reserves the collinearity.

Rotation in the Cartesian Plane:

- 1** The opposite figure shows the two images of the two points A (3 , 1) and B (− 2 , 1) by rotation R (O , 90°)

By noticing the figure , we find that :

- The image of the point A (3 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point $\hat{A} (-1 , 3)$
- The image of the point B (− 2 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point $\hat{B} (-1 , -2)$



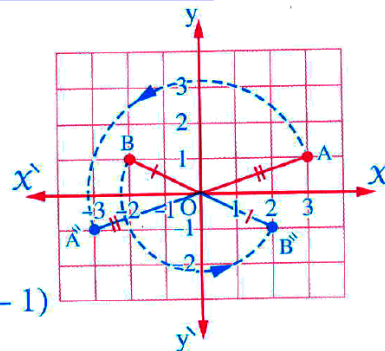
From the previous , we deduce the following rule :

The image of the point (X , y) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point (− y , X)

The opposite figure shows the two images of the two points A (3 , 1) and B (− 2 , 1) by rotation R (O , 180°)

By noticing the figure , we find that :

- The image of the point A (3 , 1) $\xrightarrow[\text{R (O , 180°)}]{\text{by rotation}}$ the point $\hat{A} (-3 , -1)$
- The image of the point B (− 2 , 1) $\xrightarrow[\text{R (O , 180°)}]{\text{by rotation}}$ the point $\hat{B} (2 , -1)$



Rotation about the origin point (O) with an angle of measure 180°

Remarks

- 1** The image of the point (X , y) $\xrightarrow[\text{R (O , - 90°)}]{\text{by rotation}}$ the point (y , − X)

For example: The image of the point (2 , − 3) $\xrightarrow[\text{R (O , - 90°)}]{\text{by rotation}}$ the point (− 3 , − 2)

- 2** Rotation about the origin point with an angle of measure 270° is equivalent to rotation about the origin point with an angle of measure (− 90°)

For example: The image of the point (2 , − 3) $\xrightarrow[\text{R (O , 270°)}]{\text{by rotation}}$ the point (− 3 , − 2)

From the previous , we deduce the following rule :

The image of the point (X, y) $\xrightarrow[\text{R (O , } 180^\circ\text{)}]{\text{by rotation}}$ the point $(-X, -y)$

Remarks

- ① The image of the point A (X, y) by rotation R $(O, 180^\circ)$ is the same image of the point A by rotation R $(O, -180^\circ)$
- ② The image of the point A (X, y) about the origin point with an angle of measure $\pm 360^\circ$ is the same point A (X, y)
- ③ Rotation with an angle of measure 90° is called a $\frac{1}{4}$ turn.
- ④ Rotation with an angle of measure 180° is called a $\frac{1}{2}$ turn.
- ⑤ Rotation with an angle of measure 360° is called the identity rotation because it returns the figure to its original position.

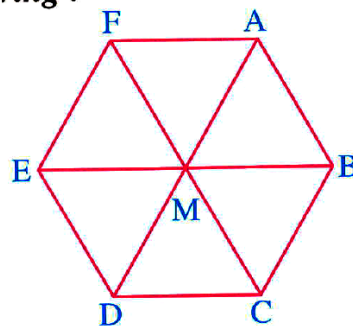
Exercises:

Complete the following table :

	The point	Its image by rotation R $(O, \pm 180^\circ)$	Its image by rotation R $(O, 90^\circ)$
1	$(3, 2)$		
2	$(-3, 4)$		
3	$(-2, -1)$		
4		$(5, -2)$	
5			$(6, 0)$

ABCDEF is a regular hexagon. Complete the following :

- ① The image of the point A by rotation around M with an angle of measure 180° is
- ② The image of $\overline{AB}$ by rotation around M with an angle of measure (-60°) is
- ③ The image of $\triangle CMD$ by rotation around M with an angle of measure 120° is



(3)	The image of the point $(2, -1)$ by reflection in X -axis is (a) $(2, 1)$ (b) $(1, 2)$ (c) $(-2, -1)$ (d) $(-1, 2)$
(4)	The image of the point $(3, -2)$ by reflection in y -axis is (a) $(2, 3)$ (b) $(-3, 2)$ (c) $(-3, -2)$ (d) $(3, 2)$
(5)	The image of the point $(-3, 0)$ is itself by reflection in (a) X -axis. (b) y -axis. (c) origin point. (d) axis of symmetry.
(6)	If $\hat{A}$ is the image of point A by reflection in M , $MA = 6$ cm. , then $AA\hat{A} = \dots\dots\dots$ cm. (a) 6 (b) 3 (c) 12 (d) 9
(7)	If the image of the point $(a - 3, 7)$ by reflection in y -axis is itself , then $a = \dots\dots\dots$ (a) 10 (b) 3 (c) -3 (d) 7
(8)	The number of axes of symmetry of the parallelogram which has a right angle equals (a) zero (b) 1 (c) 2 (d) 3
(9)	The number of axes of symmetry of the square is (a) 1 (b) 2 (c) 3 (d) 4
(10)	The number of axes of symmetry of the equilateral triangle is (a) zero (b) 1 (c) 2 (d) 3
(11)	The number of axes of symmetry of the isosceles triangle is (a) 1 (b) 2 (c) 3 (d) 4
(12)	The number of axes of symmetry of the circle equals (a) 1 (b) 2 (c) 3 (d) infinite number
(13)	The image of the point $A(3, -1)$ by translation $(1, 2)$ is (a) $\hat{A}(4, 1)$ (b) $\hat{A}(2, -3)$ (c) $\hat{A}(3, -2)$ (d) $\hat{A}(2, 3)$
(14)	The image of the point $(3, 7)$ by translation $(X + 2, y - 1)$ is (a) $(5, 6)$ (b) $(-3, 7)$ (c) $(-3, 1)$ (d) $(-1, -3)$
(15)	The image of the point $(5, -3)$ by rotation $R(O, 90^\circ)$ is (a) $(3, 5)$ (b) $(3, -5)$ (c) $(5, 3)$ (d) $(-3, -5)$

Essay questions:

Complete the following diagram :

(1)	The image of the point (2 , - 1)	by reflection in the X-axis	➡	The point (..... ,)
		by reflection in the y-axis	➡	The point (..... ,)
		by reflection in the origin point	➡	The point (..... ,)
		by translation $(X, y) \rightarrow (X - 3, y + 4)$	➡	The point (..... ,)
		by rotation R (O , 90°)	➡	The point (..... ,)
		by rotation R (O , -90°)	➡	The point (..... ,)
		by rotation R (O , $\pm 180^\circ$)	➡	The point (..... ,)
		by rotation R (O , $\pm 360^\circ$)	➡	The point (..... ,)

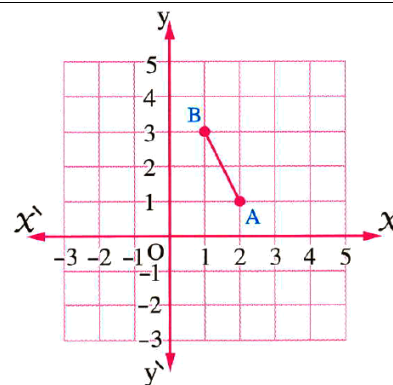
In the opposite figure :

The point A (2 , 1) and B (1 , 3)

Draw the image of $\overline{AB}$

by rotation about the origin point

with an angle of measure 90°



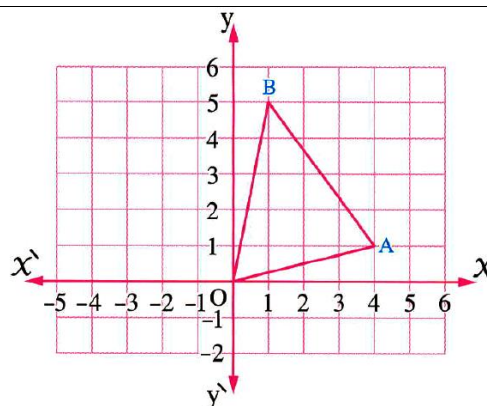
On the lattice , draw the image of $\triangle OAB$

by rotation about the origin with an angle

of measure :

1 90°

2 180°



Draw on graph paper $\triangle ABC$ where A (3 , - 1) , B (5 , 2) and C (- 2 , 4) , then draw its image by rotation about the origin point with an angle of measure 180°

On the square lattice , draw $\triangle ABC$ where A (1 , 1) , B (4 , 1) , C (5 , 4) , then draw its image by reflection in X-axis

Homework

Choose the correct answer :

- (1) The image of the point $(-1, 3)$ by translation $(4, -2)$ is
 (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$
- (2) The image of the point $(-3, 5)$ by rotation about the origin point with an angle of measure 90° is
 (a) $(-5, 3)$ (b) $(-5, -3)$ (c) $(5, 3)$ (d) $(3, -5)$
- (3) The image of the point $(-1, 3)$ by translation $(4, -2)$ is
 (a) $(3, 1)$ (b) $(3, -1)$ (c) $(5, 1)$ (d) $(5, -5)$
- (4) The number of axes of symmetry of the rectangle is
 (a) 1 (b) 2 (c) 3 (d) 4
- (5) The image of the point $(-1, 4)$ by the translation $(X, y) \longrightarrow (X + 3, y - 2)$ followed by reflection in the X-axis is
 (a) $(2, 2)$ (b) $(-2, 2)$ (c) $(-2, -2)$ (d) $(2, -2)$
- (6) The image of the point is itself by reflection in y-axis.
 (a) $(0, 3)$ (b) $(3, 0)$ (c) $(3, 3)$ (d) $(-3, 3)$

Complete the following :

- (1) The image of the point $(5, -3)$ under rotation of an angle of measure 90° about the origin point is
- (2) The image of the point $(4, -2)$ by reflection in X-axis is
- (3) The image of the point $(-2, 1)$ by rotation with an angle of measure 180° about origin point is
- (4) The image of the point $(-3, 5)$ by rotation about the origin point with an angle of measure 270° is
- (5) If the image of the point A (X, y) by a translation $(1, -4)$ is the point $(3, -2)$, then point A is (.....,)

Homework

Choose the correct answer from the given ones :

1 If the area of a rhombus is 100 square units, what is the product of the lengths of its diagonals?

- (a) 25 (b) 50
(c) 100 (d) 200

2 A rhombus has an area of 20 square feet and the length of one of its diagonals is 5 feet. What is the length of the other diagonal?

- (a) 8 feet (b) 4 feet
(c) 10 feet (d) 15 feet

3 The area of a square with a side length of 6 cm the area of a square with a diagonal length of 8 cm.

- (a) < (b) >
(c) =

4 If the area of a square is 450 square units, what is the length of its diagonal in length units?

- (a) 15 (b) 30
(c) 45 (d) 90

5 A trapezium has the sum of the lengths of its two parallel bases equal to 16 cm, and its height is 5 cm. What is its area in square centimeters?

- (a) 20 (b) 40 (c) 80 (d) 160

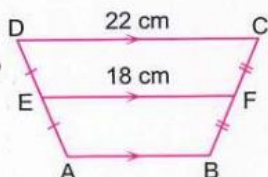
6 A trapezium the lengths of its two parallel bases are 15 cm and 11 cm. What is the length of its middle base?

- (a) 26 cm (b) 15 cm
(c) 13 cm (d) 11 cm

7 In the opposite figure:

What is the length of AB in centimeters?

- (a) 14 (b) 20
(c) 26 (d) 28



8 A trapezium has an area of 450 square inches and the lengths of its two parallel bases are 24 inches and 12 inches. What is its height?

- (a) 12.5 inches (b) 25 inches
(c) 36 inches (d) 52 inches

9 A trapezium has one of its parallel bases of length 15 cm and an area of 108 square centimeters. If its a height of length 8 cm, what is the length of the other base?

- (a) 15 cm (b) 4 cm
(c) 12 cm (d) 27 cm

10 A trapezium with a middle base length of x cm and a height is equal to half the length of its middle base. What is its area in square centimeters?

- (a) x^2 (b) $\frac{x^2}{2}$
(c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

Unit 4 - Statistics



Unit lessons:

- ① Random experiment – Sample Space - Events
- ② Theoretical and Experimental Probability

Lesson (1)

Organizing Data

Definition of random experiment

Random experiment is an experiment in which we can specify all its possible outcomes before carrying it out, but we cannot determine certainly which of them will occur.

Sample space

Sample space is the set of all possible outcomes of a random experiment and it is denoted by (S)

For example:

- ① Tossing a coin: When a coin is tossed, there are only two possible outcomes (head or tail) , then the sample space is $S = \{H, T\}$



- ② When we roll a fair die once observing the apparent number on the upper face, there are six possible outcomes, then the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

**Example ①:**

Show which of the following experiments is random and which is not, showing the number of elements of each one:

- ① When we roll a fair die once observing the apparent number on the upper face.

It is a random experiment.

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

- ② Drawing a ball from a bag contains a red ball, white ball and yellow ball.

It is a random experiment.

$$S = \{\text{red, white, yellow}\}, n(S) = 3$$

- ③ Drawing a ball from a bag contains green balls.

Not a random experiment.

- ④ Drawing a card of 7 cards numbered from 12 to 18.

It is a random experiment.

$$S = \{12, 13, 14, 15, 16, 17, 18\}, n(S) = 7$$



Show which of the following experiments is random and which is not, showing the number of elements of each one:

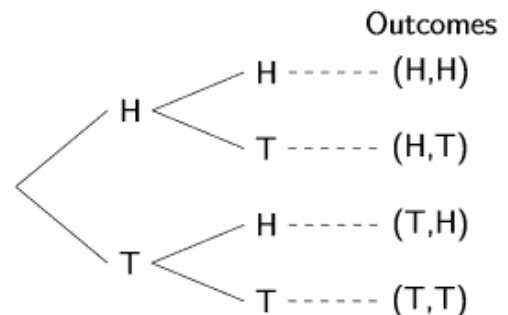
- ① Drawing a ball from a bag contains a red ball, yellow ball, white ball and green ball
- ② Throwing a cube its faces numbered from 30 to 35.
- ③ Drawing a card of 7 cards and all cards have the number 5.

Example ②:

Write the sample space showing the number of its elements:

The experiment of throwing a coin twice and observing the appearance of (head or tail).

- * Each outcome of this experiment is represented by **an ordered pair**,
its 1st coordinate is the first outcome and
its 2nd coordinate is the second outcome.



- * Using the opposite figure, we can find the sample space:

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

$$n(S) = 4$$



A restaurant serves two kinds of juices, orange (O) and mango (M), if Medhat and Noha ordered two drinks, what is all possible outcomes?

Event

Is a subset of the sample space.

For example:

If (A) is the event of appearance of an odd number when rolling a fair die once and observing the apparent number on the upper face, then $A = \{1, 3, 5\}$, $A \subset S$

- * $n(A)$: is the number of elements of the event (A)

$$n(A) = 3$$

Remarks

The certain event (S): is the event that must occur when we carry out the experiment.

The impossible event ($\emptyset$): is the event that has no chance for occurring.

The simple event: is a subset of the sample space (S) and contains only one outcome.

The possible event: is a subset of the sample space.

Example ③:

If a fair die rolled once and observing its upper face, write the sample space, then find the following events showing which of them is possible, certain and simple:

- ① Event (A) is the event of appearing an even number
- ② Event (B) is the event of appearing a number greater than 1
- ③ Event (C) is the event of appearing an even prime number
- ④ Event (D) is the event of appearing a number smaller than 7
- ⑤ Event (E) is the event of appearing the number 8

Solution:

Sample space (S) = {1, 2, 3, 4, 5, 6}

A = {2, 4, 6}

B = {2, 3, 4, 5, 6}

C = {2} Simple event

D = {1, 2, 3, 4, 5, 6} Certain event

E = $\emptyset$ Impossible event



The experiment of choosing a whole number from the numbers 2 to 11

Write the sample space, and then find the following events, and show which is a simple event, certain event and possible event.

- ① Event of appearing an even number.
- ② Event of appearing a number smaller than 6
- ③ Event of appearing a number smaller than or equal to 4
- ④ Event of appearing the number 6
- ⑤ Event of appearing an even number divisible by 9
- ⑥ Event of appearing a squared number.

Homework**① Choose the correct answer:**

- ① Drawing a card from a set of numbered cards without knowing the numbers on it.
- (a) a random experiment (b) not a random experiment
(c) impossible event (d) certain event
- ② The experiment of choosing a digit of the number 5742, what is the sample space?
- (a) {2, 4, 5} (b) {2, 4, 5, 7} (c) {57, 47, 24} (d) {5742}
- ③ The experiment of throwing a fair die once, which of the following is a simple event?
- (a) appearing a number greater than 6 (b) appearing an even prime number
(c) appearing a number smaller or equal to 2 (d) appearing an odd prime number
-

② If a fair die rolled once and observing its upper face, write the sample space, then find the following events showing which of them is possible, certain and simple:

- ① Event (A) is the event of appearing a number greater than 0
② Event (B) is the event of appearing a number divisible by 3
③ Event (C) is the event of appearing a number smaller than or equal to 4
④ Event (D) is the event of appearing a number satisfies the inequality: $x > 5$
⑤ Event (E) is the event of appearing an odd number
⑥ Event (F) is the event of appearing a number greater than 4 and smaller than 5
⑦ Event (G) is the event of appearing a number is not squared
-



③ A bag contains 25 similar cards, numbered from 1 to 25

A card is drawn randomly, write the following events:

- ① Event (A) is the event of appearing a squared number
② Event (B) is the event of appearing a number divisible by 5
③ Event (C) is the event of appearing a number smaller than 4
④ Event (D) is the event of appearing a number is a multiple of 6



Lesson (2)

Theoretical and Experimental probability

If the coach of the Egyptian national football team trained two players to take penalty kicks, the first player took 12 penalty kicks, scoring 9 goals, while the second player took 15 penalty kicks, scoring 12 goals, if the team gets the opportunity to take a penalty kick at a game. Which player does the coach choose to take the penalty kick?



In this lesson, you will learn the concept of probability and how to find its value, which will enable you to solve such life problems.

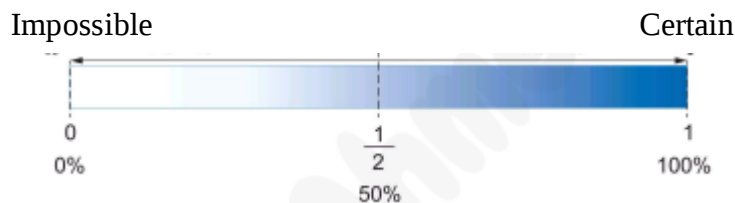
Theoretical possibility

- * Is based on the principle of equal opportunities.
- * It is equal to the ratio between the number of outcomes of the event and the total number of outcomes.
- * The probability of any event (A) = $\frac{\text{Number of outcomes of event A}}{\text{The total number of outcomes}}$

The probability of event (A) is represented by $P(A)$

Notice that

- * The probability of **an impossible** event is **zero**. Written as $P(\emptyset) = 0$
- * The probability of **a certain** event is equal to **one**. Written as $P(S) = 1$
- * The probability of **a possible** event lies between **zero** and **one**.



Example ①:

When a fair die is rolled once and the upper face is observed
Find the probability of each of the following events:



- ① (A) is the event of “appearance an even number”
- ② (B) is the event of “appearance a number less than 8”
- ③ (C) is the event of “appearance an odd prime number”
- ④ (D) is the event of “appearance the number 4”
- ⑤ (E) is the event of “appearance a number greater than 6”

Solution:

All the outcomes that appear are: 1, 2, 3, 4, 5, 6
and their number is 6

- ① The even numbers are: 2, 4, 6 and their number equals 3

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

The probability of (A) can be written as: $P(A) = 50\%$, $P(A) = 0.5$, $P(A) = \frac{1}{2}$

- ② The numbers less than 8 are: 1, 2, 3, 4, 5, 6 and their number equals 6

$$P(B) = \frac{6}{6} = 1$$

The probability of (B) can be written as: $P(B) = 100\%$, $P(B) = 1$

- ③ The odd prime numbers are: 3, 5 and their number is equal to 2

$$P(C) = \frac{2}{6} = \frac{1}{3}$$

The probability of (C) can be written as: $P(C) = 33\frac{1}{3}\%$, $P(C) = 0.\bar{3}$, $P(C) = \frac{1}{3}$

- ④ The number 4 is only one number.

$$P(D) = \frac{1}{6}$$

- ⑤ There is no number greater than 6, its number equals 0

$$P(E) = \frac{0}{6} = 0$$

The probability of (E) can be written as: $P(E) = 0\%$, $P(E) = 0$



A card is drawn randomly from identical cards numbered from 5 to 14
Find the probability of the drawn card is:

- ① An odd number.
- ② An even number greater than 9
- ③ A prime number.
- ④ A number less than 5
- ⑤ A perfect square number.

Example ②:

A coin was tossed twice and the appearance of (Head or Tail) was observed.
Find the probability of each of the following events:

- ① Event (A) “appearance of two Heads”
- ② Event (B) “appearance of a Head at least one”
- ③ Event (C) “appearance of the same thing on both shots”
- ④ Event (D) “appearance of a Head on the first shot”



Solution:

All the outcomes that appear are: (H, H) , (H, T) , (T, T) , (T, H)
and their number is 4

- ① The outcomes that have “two Heads”: (H, H) and their number is 1

$$\therefore P(A) = \frac{1}{4}$$

- ② The outcomes that have “a Head at least one”: (H, H) , (H, T) , (T, H)
and their number is 3

$$\therefore P(B) = \frac{3}{4}$$

- ③ The outcomes that have “the same thing on both shots”: (H, H) , (T, T)
and their number is 2

$$\therefore P(C) = \frac{2}{4} = \frac{1}{2}$$

- ④ The outcomes that have “a Head on the first shot”: (H, H) , (H, T)
and their number is 2

$$\therefore P(D) = \frac{2}{4} = \frac{1}{2}$$



From the following set of digits {2, 3, 5, 7}

Form a number of two different digits. If one of these numbers is chosen randomly, find the probability that it is:

- ① The tens digit is greater than the ones digit.
- ② Prime number.
- ③ One of the two numbers is even

Example ③:

A bag contains a red ball, 6 blue balls and 3 green balls, all are the similar. If a ball is drawn randomly from the bag and its colour is observed

What is the probability that the drawn ball:



- | | |
|------------------|--------------|
| ① Blue? | ② Red? |
| ③ Blue or green? | ④ White? |
| ⑤ Green? | ⑥ Not Green? |

Solution:

Let the Red ball is (R), Blue ball is (B), Green ball is (G)

The total number of the balls = 1 + 6 + 3 = 10 balls

- ① $P(B) = \frac{6}{10} = 0.6$
- ② $P(R) = \frac{1}{10} = 0.1$
- ③ $P(B \text{ or } G) = \frac{6 + 3}{10} = 0.9$
- ④ $P(W) = \frac{0}{10} = 0$
- ⑤ $P(G) = \frac{3}{10} = 0.3$
- ⑥ $P(\text{not } G) = \frac{1 + 6}{10} = 0.7$

Notice that:

The sum of the probabilities of all outcomes of any randomized experiment = 1

In example 3:

$$P(B) + P(R) + P(G) = 0.6 + 0.1 + 0.3 = 1$$

For any event A, it is:

$$P(A) + P(\text{not } A) = 1$$

Ex:

$$\text{If } P(A) = 0.4, \text{ then } P(\text{not } A) = 1 - 0.4 = 0.6$$



The cinema displays a group of films as follows:

3 comedy movies, 2 cartoon movies, 1 Horror movie, 4 Social movie.

If you choose a movie randomly,

What is the probability that the movie will be:

- ① Social?
- ② Horror movie?
- ③ Comedy or social?
- ④ Not a comedy?

Experimental probability

It depends on doing an experiment practically, then recording its outcomes (results), and then using these results to calculate **the Probability** as follows:

$$\text{The probability of event (A)} = \frac{\text{Number of times the event (A) occurred}}{\text{Number of times the experiment was performed}}$$

Example (4):

If we tossed a regular coin 100 times and the head appeared 41 times.

Find the experimental probability of the appearance of:

- ① Head (H)
- ② Tail (T)

Solution:

- ① The number of times the Head (H) appears is 41 times

$$\therefore P(H) = \frac{41}{100} = 0.41 = 41 \%$$

- ② The number of times the Tail (T) appears is: $100 - 41 = 59$

$$\therefore P(T) = \frac{59}{100} = 0.59 = 59 \%$$

Notice that

The more times the experiment is performed, the closer the value of the experimental probability is to the value of the theoretical probability.



A regular dice was thrown 150 times and the number appearing on the upper face was observed, and the results of the appearance of the numbers were as follows:

The number	1	2	3	4	5	6
No. of appearance	28	19	23	28	25	27

Find the experimental probability of:

- ① The appearance of the number 2
- ② The number 5 does not appear.
- ③ Find the theoretical probability of the appearance of the number 2

Example (5):

A rotating disc divided into several coloured sectors of equal size, if the disc is rotated 50 times, the corresponding table shows the number of times the cursor stopped on each colour.



- 1 Find the experimental probability that the cursor will stop at yellow.
- 2 Find the theoretical probability that the cursor will stop on yellow.
- 3 If the number of times the disc is rotated increases to 500 times, what do you expect about the chance that the cursor will stop at yellow?

Colour	times
Red	8
Blue	9
Yellow	13
Green	9
Purple	11

Solution:

- 1 The experimental probability of the cursor stopping at yellow is equal to:

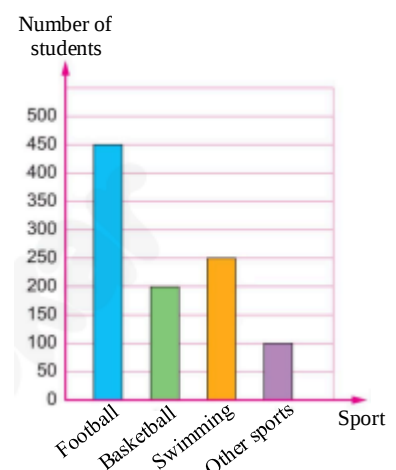
$$\frac{13}{50} = 0.26 = 26 \%$$
- 2 ∴ The five colours are evenly distributed on the disc.
 ∴ The theoretical probability that the indicator will stop at yellow is equal to:

$$\frac{1}{5} = 0.2 = 20 \%$$
- 3 When the number of times the dial is rotated increases to 500, we expect that: the chances of the cursor stopping at yellow decrease until the value of the experimental probability approaches the value of the theoretical probability (20%).



The corresponding bar chart shows the favourite sports of 1,000 students. If a student is chosen randomly:

- 1 What is the probability that he prefers basketball?
- 2 What is the probability that he does not prefer swimming?
- 3 What is the probability that he prefers football?



Example ⑥:

A Class has some students who wear glasses and other students who don't wear glasses. If a student is chosen randomly from this class and the probability that this student wears glasses is 0.1

- ① Find the probability that this student does not wear glasses.
- ② If the number of students in this class is 30 students, find the expected number of students who wear glasses.

Solution:

- ① The probability that this student does not wear glasses =
 $1 - \text{the probability that the student wears glasses} = 1 - 0.1 = 0.9$
- ② The **expected number** of outcomes of an event =
the probability of this event $\times$ the total number of all possible outcomes.
 $\therefore$ The expected number of students who wear glasses = $0.1 \times 30 = 3$ students.



- ① An experiment has 3 outcomes. If the probability of occurrence of the first outcome is 0.3 and the probability of the second is 0.45 ,
Calculate the probability of the third outcome.
- ② A bag contains 30 similar marbles. Medhat drew a marble randomly and he found it red. If the probability of drawing a red marble = $\frac{2}{5}$
find the number of red marbles in the bag.
- ③ A class contains 40 students, 30 of them succeeded in math, 24 succeeded in science and 20 succeeded in both. A student is chosen randomly.
Find the probability that this student:
 - ① Succeeded in math.
 - ② failed in science.
 - ③ Succeeded in science.
 - ④ failed in both math and science.

Home work

① Choose the correct answer:

- ① If you are thinking of buying a pen from a set of identical pens containing 5 red pens, 2 blue pens and 3 black pens and you choose a pen randomly, what is the probability that the pen is blue?
- (a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{2}{15}$ (d) $\frac{1}{15}$
- ② In the experiment of rolling a regular die once,
What is the probability of getting a number that is divisible by 2?
- (a) zero (b) $33\frac{1}{3}$ (c) 50 (d) 75
- ③ When rolling a regular die 10 times, if the number 4 appears twice on the upper face of the dice,
what is the experimental probability that the number 4 will not appear?
- (a) $\frac{1}{6}$ (b) $\frac{2}{10}$ (c) $\frac{5}{6}$ (d) $\frac{8}{10}$
- ④ If (A) is an event from a random experiment with equal chances of occurring, and the probability of event (A) is 40 % and the number of elements in the sample space is 15, so what is the number of elements?
- (a) 2 (b) 4 (c) 6 (d) 10
- ⑤ Hamza has a spinning game divided into 9 equal sections, as shown in the opposite figure. When he spins, the cursor stops randomly on one of the sections. What is the probability that the cursor stops on blue or yellow?
- (a) $\frac{4}{9}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$



- ② A box contains 80 similar balls. Some of them are red and the rest is blue.
If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

- ③ A card is chosen randomly from ten cards numbered from 1 to 10,
What is the probability that the chosen card shows :

- ① An odd number. ② An even number.
③ A prime number. ④ An odd number greater than 3

1	2	3	4	5
6	7	8	9	10

15 If $-2a^3 = -2$, what is the value of $\sqrt{a}$?

- (a) 1 (b) -1
(c) ± 1 (d) 2

16 If $-\sqrt{25} = \sqrt[3]{y}$, what is the value of y?

- (a) -5 (b) 5
(c) -125 (d) 125

17 Which of the following equals $\sqrt[3]{(8)^2}$?

- (a) -4 (b) -2
(c) 2 (d) 4

18 $|\sqrt[3]{-125}| = \sqrt{\dots\dots\dots}$

- (a) 5 (b) -5
(c) 25 (d) -25

19 If $a = 5^3$, what is the value of $\sqrt[3]{a}$?

- (a) 3 (b) 5
(c) 25 (d) 125

20 If $\sqrt{x} = 27$, what is the value of $\sqrt[3]{x}$?

- (a) 3 (b) 9
(c) 27 (d) 81

21 If $x^2 = 64$, what is the value of $\sqrt[3]{x}$?

- (a) 2 (b) -2
(c) 4 (d) ± 2

22 $\sqrt[3]{64 - \dots\dots\dots} = 3$

- (a) 9 (b) 27
(c) 37 (d) 4